Smart Waterborne Disease Control for a Scalable Population using Biodynamic Model in IoT Network

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ABSTRACT

We propose a biodynamic model for managing waterborne diseases over an Internet of Things (IoT) network, leveraging the scalability of LoRa IoT technology to accommodate a growing human population. The model, based on fractional order derivatives (FOD), enables smart prediction and control of pathogens that cause waterborne diseases using IoT infrastructure. The human-pathogen-based biodynamic FOD model utilises epidemic parameters (SVIRT: susceptibility, vaccination, infection, recovery, and treatment) transmitted over the IoT network to predict pathogenic contamination in water reservoirs and dumpsites in Iji-Nike, Enugu, the study community in Nigeria. These pathogens contribute to person-to-person, water-to-person, and dumpsite-to-person transmission of disease vectors. Five control measures are proposed: potable water supply, treatment, vaccination, adequate sanitation, and health education campaigns. A stable disease-free equilibrium point is found when the effective reproduction number of the pathogens, $\mathcal{R}_0^{eff} < 1$ and unstable if $\mathcal{R}_0^{eff} > 1$. While other studies showed a 98.2% reduction in infections when using IoT alone, this paper demonstrates that combining the SVIRT epidemic control parameters (such as potable water supply and health education campaign) with IoT achieves a 99.89% reduction in infected human populations and a 99.56% reduction in pathogen populations in water reservoirs. Furthermore, integrating treatment with sanitation results in a 99.97% reduction in infected populations. Finally, combining these five control strategies nearly eliminates infection and pathogen populations, demonstrating the effectiveness of multifaceted approaches in public health and environmental management. This study provides a blueprint for governments to plan sustainable smart cities for a growing population, ensuring potable water free from pathogenic contamination,in line with the United Nations Sustainable Development Goals #6 (Clean Water and Sanitation) and #11 (Sustainable Cities and Communities).

1. Introduction

Waterborne diseases are the diseases caused by pathogenic micro-organisms or chemical compounds that contaminate recreational or drinking water [1, 2]. According to the World Health Organisation (WHO), contaminated water and poor sanitation are linked to transmission of waterborne diseases such as cholera, diarrhoea, dysentery, hepatitis A, typhoid and polio [3]. These and other waterborne diseases are the major causes of morbidity and mortality [4]. Waterborne diseases are transmitted via directly drinking water contaminated with pathogenic organisms and are characterised by diarrhoea, which involves excessive stooling and dehydration [5, 6]. Children in developing countries are most negatively impacted by contaminated water. For example, 4.9 million children under age 5 died in 2022, and 83% of that number occurred in Sub-Saharan Africa (58%) and South Asia (25%) [7]. In this age range of children, diarrhoea disease was the second leading cause of mortality, following pneumonia and followed by malaria in 2022 [8]. While under-five deaths have reduced in other regions of the world, it has increased in sub-Saharan Africa from 31% (in 1990) to 58% (in 2022), with an expectation of further increase in the next decade due to the growing child population. According to UNICEF, Nigeria (with 844 deaths) overtook India (with 783 deaths) in under-five deaths in 2020 [8]. In 2022, Nigeria had the third highest number of affected under-five deaths in the world, with 107 deaths per 1000 live births [7, 8].

Today (in 2024), 2.4 billion people live in water-stressed countries, and hard-hit regions include Southern and Central Asia, and North Africa [9]. Along with climate change, the crisis is being fed by unchecked urbanisation, rapid population growth, pollution and land development [9]. In 2022, globally, at least 1.7 billion people use a drinking water source with microbial contamination, such as faeces [10]. By 2025, 1.8 billion people are likely to face "absolute water scarcity" and two-thirds of the global population is expected to be grappling with water stress [9]. According to the United Nations 2023 report on Goal 6 of the 17 Sustainable Development Goals, key strategies to achieve Goal 6 by 2030 is by increasing sector-wide investment, promoting innovation and enhancing cross-sectoral coordination and cooperation among all stakeholders, and adopting a more integrated and holistic approach to water management [11]. Examples of such strategies include water education, water security, climate impact education, novel water sources, mathematical

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Figure 1. Drilling (1a, 1b & 1c), implementation (2a & 2b), commissioning (3a & 3b) and ready-for-pubic-use (4a & 4b) potable water in Iji-Nike, Nigeria, funded by the IEEE I22-HAC-119 project on *Automated Borehole Design and Massive Rural Health Education in Iji-Nike Community, Enugu State.*

modelling and prediction and the use of digital technologies, such as the Internet of Things (IoT). This paper focuses on water education, biodynamic mathematical modeling and prediction, and the use of IoT to control waterborne diseases in large populations like Nigeria.

Nigerians practice distributed water supply and open dumpsite systems. Individuals provide and store water in reservoirs. Being a relatively high-temperature country with an annual average of 26.9^{o}C according to the World Bank [12], insects, mosquitoes, spiders, cockroaches and other micro-organisms enjoy the relatively cool air gaps in the tanks as suitable habitats to breed, thus contaminating the water storage systems. Pathogens from these open dumpsites can easily crawl into the water storage tanks, polluting the water. Water quality sensors can be implanted in these water tanks to measure the water quality and transmit the data through the IoT network [13]. With digital technologies, such as the water quality sensor included in water management, the health impacts can be reduced by 98% [10]. The IoT technology has also been used in smart standpipe infrastructure by Aquacheck Engineering to detect and control indiscriminate water losses [14, 15]. In that study, IoT technologies were only used to obtain and report water theft. During the COVID-19 pandemic, IoT was applied to collect data about disease symptoms and to transmit data to local health centres to quarantine patients in a planned setting [16] effectively. None of these studies considered epidemic modelling of pathogenic contamination of potable water in a scalable population in the presence of digital technologies.

As shown by WHO [17], unsafe or insufficient water sources and sanitary conditions impact a significant portion of the global population and are responsible for about 90% of deaths associated with diarrhoea. On this basis, we partnered with a health centre in Enugu, Nigeria, where Nigerians are treated when inflicted with diarrhoea disease, to study how to minimise the menace. We installed potable water funded by the IEEE-22-HAC-119 project on *Automated Borehole Design and Massive Rural Health Education in Iji-Nike Community in Enugu State* as shown in Figure 1. The main aim of this study is to investigate the efficacy of single, combined, and multiple control strategies in reducing the spread of waterborne diseases in human populations from pathogenic contamination in water reservoirs and dumpsites in the presence of digital infrastructure.

Although several models are used to study infectious diseases, fractional order derivatives (FOD) modelling is the most common approach [18, 19]. The FOD system encompasses equations of non-integer order, including Caputo, Riemann–Liouville, Atangana-Baleanu, and Caputo–Fabrizio derivatives [20, 21, 22]. FOD systems provide more accurate modelling approaches than traditional integer-based differential equations [23]. For instance, fractional calculus, such as employing Caputo and Atangana–Baleanu–Caputo (ABC) fractional derivatives has been used to investigate bovine babesiosis disease [23, 24]. Memory effects is a significant advantage of FOD models. These models are multistage, capture factuality, memory effects, and provide superior data fitting compared to integer-order models, which often struggle to account for system memory and hereditary properties. Beyond health studies, FOD is used in finance [25], mechanics [26], image processing [27], physics, and engineering [28].

Our study models an unplanned community setting protracted with waterborne disease but with the data sent to a local healthcare centre through an IoT infrastructure. The unplanned setting involves two population styles, namely

human and pathogen populations. We aim to contribute to at least two of the 17 United Nations Sustainable Development Goals: #6 on Clean Water and Sanitation and #11 on Sustainable Cities and Communities towards potable water using IoT and computational modelling. We model a smart water infrastructure system (using FOD) that combines the waterborne disease and pathogenic contamination parameters that are sent over an IoT network to investigate, plan, predict and control the spread of the disease vector.

On top of that, the FOD SEIR (Susceptible - Exposed - Infectious - Recovered) epidemic model was used to model infectious disease transmission dynamics, using Caputo-type fractional derivatives as fractional operators [23]. Fractional optimal control problems have also been used in real cholera outbreak cases [29, 30]. In all these studies, the results showed that FOD models achieved more accurate approximations than integer-order models. It follows that FOD models are more efficient than traditional integer-order ones in modelling health-based systems and will be used in studying other health-based systems, such as pathogenic contamination of water and waterborne diseases in scalable populations with the aid of IoT network. A major limitation in using FOD models in epidemic problems both in (SVIRT and SEIR) is in the mathematical Complexity, for example, fractional order models for smart waterborne diseases can be complex and challenging to apply in cases with limited data thus requiring more advanced mathematical techniques.

In this paper, our main contributions are:

- formulated the spread of waterborne disease in an unplanned setting (i.e., scalable human population) into a mathematical model using FOD. We refer to this model as a biodynamic model of waterborne disease spread covering spread situations such as person-to-person, water-to-person and dumpsite-to-person;
- adapt well-established epidemic parameters, SVIRT, in the study and these parameters are transmitted over an IoT infrastructure that suits scalable population;
- Although UNICEF identifies two control variables for waterborne disease, this study explores five control variables towards eradicating waterborne disease in a scalable human population. Our multivariable control parameters show that waterborne disease can be significantly reduced with two of the variables;
- derive the waterborne disease-free equilibrium and stability conditions. On top of that, we develop epidemiologically and biologically feasible regions in the waterborne disease dynamics.
- built and deployed a potable water supply service to the Iji-Nike Community in Enugu, Nigeria, as an example of a control measure for waterborne disease. Our study validates waterborne disease controls by leveraging suitable potable water supply and consumption, health education campaigns, sanitation, vaccination, and treatment. In this case, we tested for the optimal control of waterborne disease on our proposed biodynamic model by combining all five strategies and determined the control efficiency;
- our model can be used by local and national governments for planning new sustainable smart cities free from waterborne diseases. Additionally, the model and infrastructure can be used to predict and control the spread of waterborne disease in an existing community.

The remaining parts of the paper are organised as follows. Section 2 discusses the system model and Section 3 presents the stability of the disease equilibrium conditions of the model. The fractional optimal control of the smart waterborne disease model is discussed in Section 4, and the simulation results are discussed in Section 5. Results and Discussions are presented in Section 6 with the conclusion in Section 7.

2. System Model Formulation

This study presents a biodynamic model of waterborne disease that involves human and pathogen populations. The human population is made up of five compartments, namely, Susceptible $S(t)$, Vaccinated $V(t)$, Infected $I(t)$, Treated $T(t)$ and Recovered $R(t)$. The pathogen population has two compartments, that is, pathogen populations in water reservoir $W(t)$ and Dumpsite $D(t)$. The waterborne disease model is a combination of system of human population and the pathogen population in water reservoir and dumpsite. This study extends the $SVITR$ model under spatiotemporal human population size $N(t)$ by adding two compartments $W(t)$ and $D(t)$ that measure pathogen concentration in water reservoir and dumpsite respectively. These parameters are transmitted over IoT infrastructure.

Close proximity to an open dumpsite poses a significant risk of waterborne disease transmission as individuals may inadvertently come into direct contact with contaminated waste, leading to potential infection [31]. Also, pathogens in open dumpsites can get into water sources through leachate and contaminated unprotected groundwater sources or through precipitation that washes into surface water [32, 33]. Improving the supply of clean potable water and sanitation services is an effective means of lowering the number of infected people and the number of people who die each year because of waterborne diseases such as cholera, Typhoid fever, Dysentery, Giardia, etc. [34, 35]. An infected individual generates secondary infections through direct contact with susceptible individuals and by shedding the pathogen into a water body, which other susceptible individuals subsequently encounter. In Iji-Nike Enugu, we provided potable water that is safe and free from pathogenic contaminations by drilling boreholes – this was funded by the IEEE 22-HAC-119 project on Automated Borehole Design and Massive Rural Health Education in Iji-Nike Community in Enugu State, (see Figure 1).

Considering Figure 2, the nonlinear ordinary differential equations are formed based on these assumptions:

- The total population of individuals is not constant; thus, the susceptible population are recruited at constant rate ψ.
- Controls are implemented by continuous provision and consumption of potable water.
- Vaccination is applied to the susceptible population.
- Susceptible population can be infected by infected human, infected water and contact with open dumpsite.
- Infected persons receive therapeutic treatment at a rate σ, while the pathogens in water reservoir are reduced

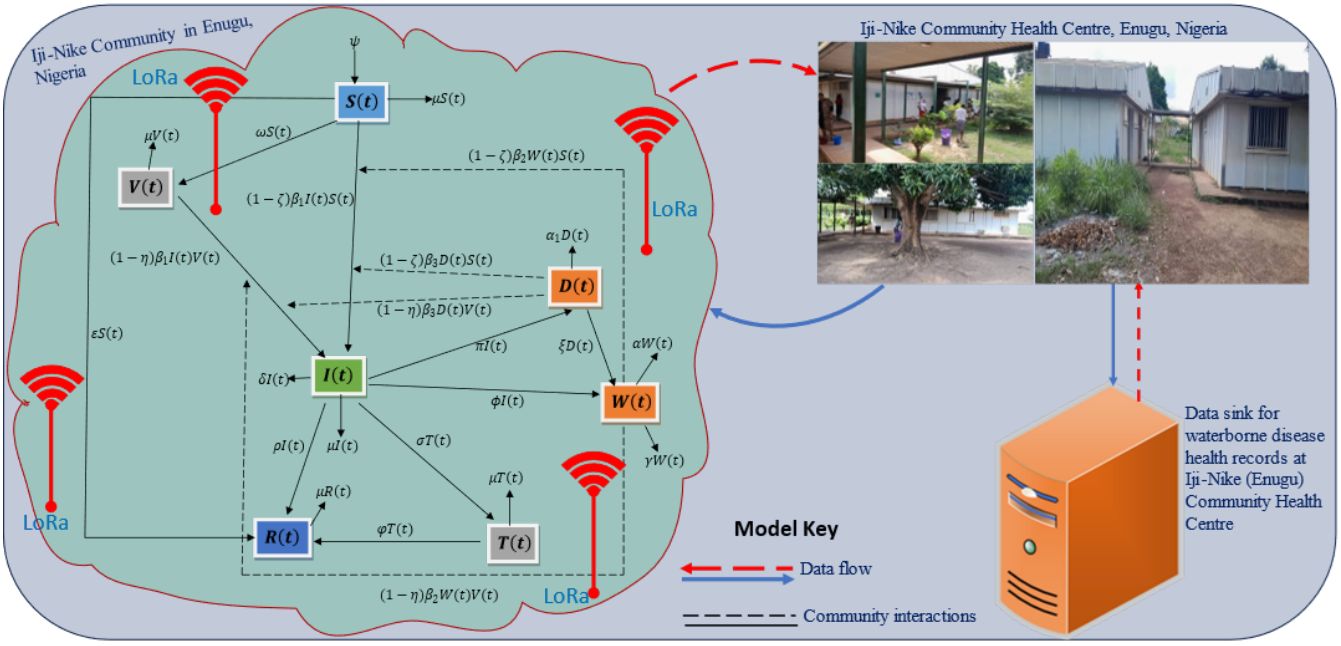

Figure 2. Biodynamic model of waterborne diseases spread and control in Iji-Nike Community, Enugu, Nigeria. The model shows the interactions of human and pathogen populations in the Iji-Nike Community and these parameters are transmitted over an IoT Network for data analysis and control of the spread of the disease vector.

via adequate provision of potable water at a rate α and in dumpsite via proper sanitation at a rate α_1.

- Individuals in each compartment have the same natural death rate μ, and infected individuals die due to the infection at a rate δ.
- Recovered individuals obtain temporary immunity.
- Water treatment and dumpsite sanitation lead to the death of pathogens [36].

Figure 2 depicts a network of Susceptible $S(t)$, Vaccinated $V(t)$, Infected $I(t)$, Treated $T(t)$ and Recovered $R(t)$ persons in the presence of $W(t)$ and $D(t)$ parameters that measure pathogen concentration in water reservoir and dumpsite connected. These are transmitted via an IoT infrastructure such as LoRaWAN to the isolated server of the Iji-Nike hospital for waterborne disease study. The model can be extended using any other LPWAN technology such as the NB-IoT, Sigfox, RPMA, WavIoT and LTE-M [37]. This study adopts LoRaWAN due to the tripartite advantages of wide coverage, low power consumption and low data rate [38]. These parameters, especially the scalability of LoRaWAN, present useful advantages since the proposed model assumes a non-constant population. This study was carried out in the Iji-Nike Community in Enugu, Nigeria, as depicted in Figure 2. Due to topology constraints, four LoRa stations are distributed over the Iji-Nike Enugu community to transmit waterborne disease data sent to an independent server local in the area. The data are received through some edge control systems within the health centre premises.

2.1. Infrastructure Model Integer Derivative

To begin with, we approach the problem as a dynamic system. Dynamic systems comprise a set of equations that articulate natural events, primarily focusing on time-evolving processes [39]. By combining assumptions, definitions, variable and parameter descriptions, and the schematic (transfer) diagram depicted in Figure 2, this study presents the biodynamic model of waterborne diseases as a system of integer derivatives (as shown in (1)): where $\mathcal{X} = (\delta + \mu + \rho + \sigma)$.

Table 1: State Variables of Waterborne Diseases Model, each depends on time, t.

State Variables	Description
$S(t)$	Susceptible human population
$V(t)$	Vaccinated human population
$I(t)$	Infected human population
$T(t)$	Treated human population
$R(t)$	Recovered human population
$W(t)$	Pathogen population in water reservoir
$D(t)$	Pathogen population in Dumpsite

We define the initialised boundary conditions as $S(0) = S^0 \geq 0$, $V(0) = V^0 \geq 0$, $I(0) = I^0 \geq 0$, $T(0) = T^0 \geq 0$, $R(0) = R^0 \geq 0$, $W(0) = W^0 \geq 0$, $D(0) = D^0 \geq 0$, where $\gamma = \theta - \nu$ and the populations under study are characterised as:

$$N_1(t) = S(t) + V(t) + I(t) + T(t) + R(t) \quad (2a)$$

$$N_2(t) = W(t) + D(t). \quad (2b)$$

Notice that population (2a) is the human population while population (2b) is the pathogen population.

2.2. Caputo Fractional order Model

Based on the integer order model in (1), we derive a non-integer order model and consider the saturation of both pathogenicity rate and saturated provision and consumption of potable water, with awareness, treatment function, effective vaccination and adequate water supply. Such model does

$$\frac{dS(t)}{dt} = \psi - (1-\zeta)\left(\beta_1 I(t) + \beta_2 W(t) + \beta_3 D(t)\right) S(t) - (\mu + \omega + \varepsilon) S(t) \tag{1a}$$

$$\frac{dV(t)}{dt} = \omega S(t) - (1-\eta)\left(\beta_1 I(t) + \beta_2 W(t) + \beta_3 D(t)\right) V(t) - \mu V(t) \tag{1b}$$

$$\frac{dI(t)}{dt} = (1-\zeta)\left(\beta_1 I(t) + \beta_2 W(t) + \beta_3 D(t)\right) S(t) + (1-\eta)\left(\beta_1 I(t) + \beta_2 W(t) + \beta_3 D(t)\right) V(t) - \mathcal{X} I(t) \tag{1c}$$

$$\frac{dT(t)}{dt} = \sigma I(t) - (\mu + \varphi) T(t) \tag{1d}$$

$$\frac{dR(t)}{dt} = \varphi T(t) + \rho I(t) + \varepsilon S(t) - \mu R(t) \tag{1e}$$

$$\frac{dW(t)}{dt} = \phi I(t) + \xi D(t) - (\gamma + \alpha) W(t) \tag{1f}$$

$$\frac{dD(t)}{dt} = \pi I(t) - \left(\xi + \alpha_1\right) D(t). \tag{1g}$$

Table 2: Nomenclature of variable in the biodynamic model for waterborne diseases spread

Term	Description
ψ	Recruitment rate into susceptible compartment
β_1	Disease transmission coefficient for person to person (person to person contact rate)
β_2	Disease transmission coefficient for water to person (water to person contact rate)
β_3	Disease transmission coefficient for dumpsite to person (dumpsite to person contact rate)
η	Vaccine efficacy
ϕ	Rate at which infected individuals shed pathogen into water
ω	Vaccination rate
μ	Natural death rate
φ	Recovery rate after treatment
σ	Treatment rate
ε	Potable water provision and consumption rate
α	Adequate potable (clean) water supply
ρ	Rate at which the infected individuals recover naturally due to immune response
ζ	Rate at which humans became aware of the waterborne diseases via health education campaigns and join the recovered class
γ	Natural decay rate of pathogens in water reservoir
θ	Natural generation rate of pathogens in water
ν	Decay rate of pathogen in water
δ	Death rate of the infected individuals due to waterborne diseases
α_1	Adequate sanitation management services
ξ	Dumpsite to water reservoir contact rate
π	Rate at which human activities increase pathogen in dumpsites.

not include memory effect which is required for describing biodynamic model. The Atangana – Baleanu Caputo (ABC) operator introduced by Atangana and Baleanu, [40] explained the memory effect concisely. We then define the model with ABC operator ${}_0^{ABC}D_t^q$ as (3); where $S(0) = S^0$, $V(0) = V^0$, $I(0) = I^0$, $T(0) = T^0$, $R(0) = R^0$, $W(0) = W^0$ and $D(0) = D^0$ are the initial conditions. Also, $\mathcal{X} = (\delta + \mu + \rho + \sigma)$ and $\xi_{\alpha_1} = \left(\xi + \alpha_1\right)$.

Preliminaries This section will cover the fundamentals of fractional derivatives [41, 42], which are an extension of integer derivatives. The Riemann Liouville and Caputo fractional derivatives are defined in this work, beginning with the Liouville-Caputo fractional derivative.

Definition 1. *The fractional derivative of the Atangana-Baleanu type in the Liouville-Caputo (ABC) sense was defined by Abdon and Baleanu [21] as*

$${}_0^{ABC}D_t^q\{f(t)\} = \frac{ABC(q)}{1-q}\int_0^t f'(p) E_q\left[-q\frac{(t-p)}{1-q}\right] dp,$$

where $0 < q < 1$ *.*$ABC(q)$ *is the normalisation constant that satisfies* $ABC(0) = ABC(1) = 1$*. The one parameter Mittag – Leffter function* $E_q[z]$ *is defined as*

$$E_q[z] = \sum_{h=0}^{\infty} \frac{z^h}{\Gamma(qh+1)},$$

where $z \in \mathbb{C}$, $\mathbb{R}(q) > 0$, $\Gamma(\cdot)$ *is the function,*

$$\Gamma(\mathrm{x}) = \int_0^{+\infty} t^{x-1} e^{-t} dt$$

and satisfies $\Gamma(\mathrm{x}+1) = \mathrm{x}\Gamma(\mathrm{x})$.

Definition 2. *Let us consider the Riemann- Liouville fractional order integer* $(g\ order)$ *definition as*

$${}_{t_0}^{RL}D_t^{-g} f(t) = \frac{1}{\Gamma(\mathrm{g})}\int_{t_0}^t (t-\tau)^{g-1} f(\tau)\, d\tau, \quad g > 0.$$

Definition 3. *Definition 3: The Riemann-Liouville fractional derivative's Laplace transform, as shown in [43], is given by* (4)

$$L\left\{{}_0^{RL}D_t^g f(t); S\right\} = S^g F(S) - \sum_{h=0}^{n-1} S^h\left[{}_{t_0}^{RL}D_t^{g-h-1} f(t)\right]_{t=0'}, \tag{4}$$

where $n-1 \le g < n$.

Definition 4. *The Laplace transform of the Caputo fractional derivative is given in [33] as*

$$\mathcal{L}\left\{{}_0^{RL}D_t^g f(t); S\right\} = S^g F(S) - \sum_{h=0}^{n-1} S^{g-h-1} f^{(h)}(t) \tag{5}$$

$$
\begin{aligned}
&{}_0^{ABC}D_t^q S(t) = \psi - (1-\zeta)\left(\beta_1 I(t) + \beta_2 W(t) + \beta_3 D(t)\right) S(t) - (\mu+\omega+\varepsilon) S(t) && (3a)\\
&{}_0^{ABC}D_t^q V(t) = \omega S(t) - (1-\eta)\left(\beta_1 I(t) + \beta_2 W(t) + \beta_3 D(t)\right) V(t) - \mu V(t) && (3b)\\
&{}_0^{ABC}D_t^q I(t) = (1-\zeta)\left(\beta_1 I(t) + \beta_2 W(t) + \beta_3 D(t)\right) S(t) + (1-\eta)\left(\beta_1 I(t) + \beta_2 W(t) + \beta_3 D(t)\right) V(t) - \mathcal{X} I(t) && (3c)\\
&{}_0^{ABC}D_t^q T(t) = \sigma I(t) - (\mu+\varphi) T(t) && (3d)\\
&{}_0^{ABC}D_t^q R(t) = \varphi T(t) + \rho I(t) + \varepsilon S(t) - \mu R(t) && (3e)\\
&{}_0^{ABC}D_t^q W(t) = \phi I(t) + \xi D(t) - (\gamma+\alpha) W(t) && (3f)\\
&{}_0^{ABC}D_t^q D(t) = \pi I(t) - \xi_{\alpha_1} D(t), && (3g)
\end{aligned}
$$

where $n-1 < g \le n$.

Definition 5. *Consider the Caputo fractional Derivative*

$$
{}_0^{ABC}D_t^q f(t) = k(t,\ f(t)),\ q \in (0,\ 1). \tag{6}
$$

Then r^* *is the equilibrium point of system* (6) *if and only if* $k(t,\ r^*) = 0$. *Therefore,*

$$
{}_0^{C}I_t^q = \mathcal{U}(t) = k(t,\ \mathcal{U}(t)),\ q \in (0,\ 1),
$$

if and only if $k(t,\ \mathcal{U}^*)$.

2.3. Invariant Region and Boundedness

From the model system (3), we investigate the invariant region, which is significant in dynamic systems. Thus, we shall explore the dynamic analysis of the model in the feasible region Ω having in mind that the solution on Ω is still in Ω. Remember that our model system (3) contains two parts, that is, the human and the pathogen parts which then give the feasible region as

$$
\Omega = \Omega_{\mathrm{H}} \cup \Omega_{\mathrm{W}} \cup \Omega_{\mathrm{D}} \subset \mathbb{R}_+^5 \times \mathbb{R}_+^1 \times \mathbb{R}_+^1, \tag{7}
$$

which can be simplified into

$$
\Omega_{\mathrm{H}} = \left\{ \mathrm{S}(t) + \mathrm{V}(t) + \mathrm{I}(t) + \mathrm{T}(t) + \mathrm{R}(t) \in \mathbb{R}_+^5 \le \frac{\psi}{\mu} \right\} \tag{8a}
$$

$$
\Omega_{\mathrm{W}} = \left\{ W(t) \in \mathbb{R}_+^1 \le \frac{\psi\left(\phi\xi + \phi\alpha_1 + \xi\pi\right)}{\mu\left(\xi+\alpha_1\right)(\gamma+\alpha)} \right\} \tag{8b}
$$

$$
\Omega_{\mathrm{D}} = \left\{ D(t) \in \mathbb{R}_+^1 \le \frac{\psi\pi}{\mu\left(\xi+\alpha_1\right)} \right\}. \tag{8c}
$$

To obtain an invariant region, at which the model can be considered to be epidemiologically and mathematically well-posed, we sum (3a), (3b), $\cdots$, (3e) to get

$$
{}_0^{ABC}D_t^q N_g(t) = \psi - \mu N_g(t) - \delta I(t).
$$

Observe that at disease-free equilibrium, $\delta I(t) = 0$, thus

$$
{}_0^{ABC}D_t^q N_g(t) = \psi - \mu N_g(t) \tag{9}
$$

Using Laplace transform in (9) gives

$$
\begin{aligned}
&\mathcal{L}\left[{}_0^{ABC}D_t^q N_g(t) + \mu N_g(t)\right] = \mathcal{L}[\psi]\\
&N_g(s) \le N_g(0)\frac{s^{g-1}}{s^g+\mu} + \frac{\psi}{s\left(s^g+\mu\right)}. \qquad (10)
\end{aligned}
$$

Using the inverse Laplace transform, we complete system (10) to obtain

$$
N_g(t) \le \left(N_g(0) - \frac{\psi}{\mu}\right) E_{q,\,1}(-\mu t^q) + \frac{\psi}{\mu}, \tag{11}
$$

where $N_g(0)$ is the initial condition. Therefore, $0 \le N_g(t) \le \frac{\psi}{\mu}$ *as* $t \to \infty$ and hence Ω is positively invariant and attractive set in $\mathbb{R}_+^5$. Furthermore, since for all $t > 0$ satisfying the Mittag – Leffler function is bounded and

$$
\lim_{t\to\infty} E_{q,\,1}(-\mu t^q) = 0. \tag{12}
$$

Then,

$$
N_g(t) \le \frac{\psi}{\mu}. \tag{13}
$$

Substituting (13) into (3d), we obtain

$$
{}_0^{ABC}D_t^q D(t) = \pi\frac{\psi}{\mu} - \xi_{\alpha_1} D(t) \le \pi\frac{\psi}{\mu} - \xi_{\alpha_1} D(t). \tag{14}
$$

After the Laplace transform and inverse Laplace transform on the above formula, we obtain

$$
\begin{aligned}
D(t) &\le \left(D(0) - \frac{\pi\psi}{\mu\xi_{\alpha_1}}\right) E_q\left(-\xi_{\alpha_1} t^q\right) + \frac{\pi\psi}{\mu\xi_{\alpha_1}} && (15a)\\
&\le \frac{\pi\psi}{\mu\left(\xi+\alpha_1\right)}. && (15b)
\end{aligned}
$$

Also, by substituting (13) and (15) into (3e), we obtain

$$
\begin{aligned}
{}_0^{ABC}D_t^q D(t) &= \phi\frac{\psi}{\mu} + \xi\frac{\pi\psi}{\mu\xi_{\alpha_1}} - (\gamma+\alpha) W(t)\\
&\le \frac{\psi\phi}{\mu} + \frac{\xi\pi\psi}{\mu\xi_{\alpha_1}} - (\gamma+\alpha) W(t).
\end{aligned}
$$

After Laplace transform and inverse Laplace transform on the above formula, then

$$
\begin{aligned}
W(t) \le &\left(W(0) - \frac{\psi\left(\xi\phi + \phi\alpha_1 + \xi\pi+\right)}{\mu\xi_{\alpha_1}(\gamma+\alpha)}\right) E_q\left(-(\gamma+\alpha)t^q\right) +\\
&\frac{\psi\left(\xi\phi + \phi\alpha_1 + \xi\pi+\right)}{\mu\left(\xi+\alpha_1\right)(\gamma+\alpha)},
\end{aligned}
$$

which can be rewritten as

$$
W(t) \le \frac{\psi\left(\xi\phi + \phi\alpha_1 + \xi\pi+\right)}{\mu\left(\xi+\alpha_1\right)(\gamma+\alpha)}. \tag{16}
$$

$$\Omega = \left(S(t), V(t), I(t), T(t), R(t), W(t), D(t)\right) \in \mathbb{R}_+^7 : S(t)+V(t)+I(t)+T(t)+R(t) \leq \frac{\Psi}{\mu} \tag{17a}$$

$$W(t) \leq \frac{\psi\left(\phi\xi+\phi\alpha_1+\xi\pi\right)}{\mu\left(\xi+\alpha_1\right)(\gamma+\alpha)}, \tag{17b}$$

$$D(t) \leq \frac{\pi\psi}{\mu\left(\xi+\alpha_1\right)} \tag{17c}$$

Thus, we can conclude that Ω is the positive invariant region of (3) and the solutions is bounded and we have (17):

2.4. Existence and Positivity of the Solution

Theorem 1. *The solution of the model system* (3) *is positive, provided the initial conditions are positive for all* $t > 0$.

Proof. For the positivity of the solution of (3), we shall recall the generalised mean value theorem by [44]. □

Lemma 1. *Let* $\mathcal{H}(y) \in C\left[b_1, b_2\right]$ *and* ${}_l^C\mathcal{D}_t^\gamma \mathcal{H}(y) \in C\left(b_1, b_2\right]$, *then,*

$$\mathcal{H}(t) = \mathcal{H}\left(b_1\right) + \frac{1}{\Gamma(\gamma)}\left({}_l^C\mathcal{D}_t^\gamma \mathcal{H}\right)(\vartheta)\left(t-b_1\right)^\gamma$$

where, $b_1 \leq \vartheta \leq t \; \forall \; t \in \left(b_1, b_2\right]$.

Proposition 1. *Suppose* $\mathcal{H}(y) \in C\left[b_1, b_2\right]$ *and* ${}_l^C\mathcal{D}_t^\gamma \mathcal{H}(y) \in C\left(b_1, b_2\right] \; \forall \; \gamma \in (0, 1]$, *then if*

1. ${}_l^C\mathcal{D}_t^\gamma \mathcal{H}(y) \geq 0 \; \forall \; y \in \left(b_1, b_2\right)$, *then* $\mathcal{H}(y)$is non decreasing in the interval $\left[b_1, b_2\right]$.
2. ${}_l^C\mathcal{D}_t^\gamma \mathcal{H}(y) \leq 0 \; \forall \; y \in \left(b_1, b_2\right)$, *then* $\mathcal{H}(y)$is non-increasing in the interval $\left[b_1, b_2\right]$.

Theorem 2. *A unique solution of system* (3) *exists and will remain in the region* $\mathbb{R}_+^7$ *and will be positive for all* $t > 0$.

Proof. From Lin [45], we prove the existence of the proposed waterborne diseases model system (3). □

To guarantee that the solution will always be positive, it is sufficient to demonstrate that on each hyperplane bounding the positive orthant, the vector field points to $\mathbb{R}_+^7$. Then, system (3) leads to

$${}_0^{ABC}D_t^q S(t) = \psi \geq 0 \tag{18a}$$

$${}_0^{ABC}D_t^q V(t) = \omega S(t) \geq 0 \tag{18b}$$

$${}_0^{ABC}D_t^q I(t) = (1-\zeta)\left(\bar{h}\right) S(t) + (1-\eta)\left(\bar{h}\right) V(t) \geq 0 \tag{18c}$$

$${}_0^{ABC}D_t^q T(t) = \sigma I(t) \geq 0 \tag{18d}$$

$${}_0^{ABC}D_t^q R(t) = \varphi T(t) + \rho I(t) + \varepsilon S(t) \geq 0 \tag{18e}$$

$${}_0^{ABC}D_t^q W(t) = \phi I(t) + \xi D(t) \geq 0 \tag{18f}$$

$${}_0^{ABC}D_t^q D(t) = \pi I(t) \geq 0. \tag{18g}$$

where $\bar{h} = \beta_1 I(t)+\beta_2 W(t)+\beta_3 D(t)$. Thus, using the above Proposition 1, it can be seen deduced that all solutions will be in $\mathbb{R}_+^7$. This implies that

3. Disease-Free and Endemic Equilibrium Points

The waterborne diseases model system (3) has a disease-free equilibrium E^0. For example, we set the right-hand side equal to zero, and we obtain (20).

The endemic equilibrium, which is the point at which the pathogen cannot totally be eradicated from the population, is computed by setting the right-hand side of (3) to zero so that,

$$D^{**} = \frac{\pi I^{**}}{\left(\xi+\alpha_1\right)} \tag{21a}$$

$$W^{**} = \frac{\phi\left(\xi+\alpha_1\right) I^{**} + \xi\pi I^{**}}{\left(\xi+\alpha_1\right)(\gamma+\alpha)} \tag{21b}$$

$$T^{**} = \frac{\sigma I^{**}}{(\mu+\varphi)} \tag{21c}$$

$$R^{**} = \frac{\varphi\sigma I^{**} + \rho(\mu+\varphi) I^{**} + \varepsilon(\mu+\varphi) S^{**}}{\mu(\mu+\varphi)} \tag{21d}$$

Let

$$\begin{aligned}\beta_1 I^{**} + \beta_2 W^{**} + \beta_3 D^{**} = \beta_1 I^{**} + \cdots \\ \beta_2\left(\frac{\phi\xi_{\alpha_1} I^{**} + \xi\pi I^{**}}{\xi_{\alpha_1}(\gamma+\alpha)}\right) + \beta_3\left(\frac{\pi I^{**}}{\xi_{\alpha_1}}\right)\end{aligned} \tag{22}$$

$$\Longrightarrow \frac{\left[\bar{\beta}_1(\gamma+\alpha) + \beta_2\left(\phi\xi_{\alpha_1} + \xi\pi\right) + \beta_3\pi(\gamma+\alpha)\right] I^{**}}{\xi_{\alpha_1}(\gamma+\alpha)}.$$

where $\bar{\beta}_1 = \beta_1\left(\xi+\alpha_1\right) = \beta_1\xi_{\alpha_1}$.

If we let $A = \beta_1\left(\xi+\alpha_1\right)(\gamma+\alpha)+\beta_2\left(\phi\left(\xi+\alpha_1\right)+\xi\pi\right)+\beta_3\pi(\gamma+\alpha)$, $B = \xi+\alpha_1$, $C = \gamma+\alpha$, $E = \mu+\varphi$, $G = \mu+\omega+\varepsilon$ and $H = (\delta+\mu+\rho+\sigma)$, then

$$\begin{aligned}(1-\zeta)(1-\eta)A^2 H I^{**2} + \big[ABCH(\mu(1-\zeta)+(1-\eta)G) \\ - \psi(1-\zeta)(1-\eta)A^2\big] I^{**} + BC\big[\mu BCGH - \\ \psi A[\mu(1-\zeta)-\omega(1-\eta)]\big] = 0,\end{aligned}$$

which can be rewritten in the form:

$$a_2 I^{**2} + a_1 I^{**} + a_0 = 0, \tag{23}$$

where

$$a_2 = (1-\zeta)(1-\eta)A^2 H \tag{24a}$$

$$a_1 = ABCH(\mu(1-\zeta)+(1-\eta)G) - \psi(1-\zeta)(1-\eta)A^2 \tag{24b}$$

$$a_0 = BC[\mu BCGH - \psi A[\mu(1-\zeta)-\omega(1-\eta)]]. \tag{24c}$$

$$\chi \in \mathbb{R}_+^7 : \chi \geq 0. \tag{19}$$

where $\chi = \left({}_0^{ABC}D_t^q S(t), {}_0^{ABC}D_t^q V(t), {}_0^{ABC}D_t^q I(t), {}_0^{ABC}D_t^q T(t), {}_0^{ABC}D_t^q R(t), {}_0^{ABC}D_t^q W(t), {}_0^{ABC}D_t^q D(t)\right)$.

$$E^0 = \left(S^0,\ V^0,\ I^0,\ T^0,\ R^0,\ W^0,\ D^0\right) = \left(\frac{\psi}{\mu+\omega+\varepsilon}, \frac{\psi\omega}{\mu(\mu+\omega+\varepsilon)},\ 0,\ 0,\ \frac{\psi\varepsilon}{\mu(\mu+\omega+\varepsilon)},\ 0,\ 0\right). \tag{20}$$

Therefore, letting E^{**} be the waterborne diseases endemic equilibrium of the model, we have

$$E^{**} = \left(S^{**}, V^{**}, I^{**}, T^{**}, R^{**}, W^{**}, D^{**}\right), \tag{25a}$$

where:

$$S^{**} = \frac{\psi BC}{((1-\zeta)AI^{**} + BCG)} \tag{25b}$$

$$V^{**} = \frac{\omega\psi(BC)^2}{((1-\zeta)AI^{**} + BCG)\left[(1-\eta)AI^{**} + \mu BC\right]} \tag{25c}$$

$$\left(1-\zeta\right)(1-\eta)A^2HI^{**2} + \left[ABCH\left(\mu(1-\zeta) + \cdots\right.\right.$$
$$\left.(1-\eta)G\right) - \psi(1-\zeta)(1-\eta)A^2\Big]I^{**} + BC\Big[\mu BCGH \cdots$$
$$\left. - \psi A\left[\mu(1-\zeta) - \omega(1-\eta)\right]\right] = 0 \tag{25d}$$

$$T^{**} = \frac{\sigma I^{**}}{(\mu+\varphi)} \tag{25e}$$

$$R^{**} = \frac{((1-\zeta)AI^{**} + BCG)I^{**}(\varphi\sigma + \bar{\rho}) + \bar{\psi}}{\mu(\mu+\varphi)((1-\zeta)AI^{**} + BCG)} \tag{25f}$$

$$W^{**} = \frac{\phi\left(\xi+\alpha_1\right)I^{**} + \xi\pi I^{**}}{\left(\xi+\alpha_1\right)(\gamma+\alpha)} \tag{25g}$$

$$D^{**} = \frac{\pi I^{**}}{\left(\xi+\alpha_1\right)}. \tag{25h}$$

where $\bar{\rho} = \rho(\mu+\varphi)$ and $\bar{\psi} = \psi\varepsilon(\mu+\varphi BC)$.

In our biodynamic model, the basic reproduction number, (i.e., the anticipated number of secondary cases resulting from a single infection in a population that is fully susceptible [46], is crucial for assessing the endemicity of illnesses. We used the next generation operator technique [47] in (3) to determine the basic reproduction number, $\mathcal{R}_0$. The $\mathcal{R}_0$ is obtained by taking the largest (dominant) eigenvalue (spectral radius) of

$$FV^{-1} = \left[\frac{\partial F_i\left(E^0\right)}{\partial x_j}\right]\left[\frac{\partial V_i\left(E^0\right)}{\partial x_j}\right]^{-1}, \tag{26}$$

where F_i is the rate of appearance of new infection in node i, V_i is the transfer of infection from one node i to another and E^0 is the disease-free equilibrium [36]. From model system (3), we rewrite the equations with infection and pathogen, that is, $I(t), W(t)$ and $D(t)$. This leads to

$${}_0^{ABC}D_t^q I(t) = (1-\zeta)\left(\bar{h}\right)\left(S(t) + (1-\eta)V(t)\right) - \mathcal{X}I(t) \tag{27a}$$

$${}_0^{ABC}D_t^q W(t) = \phi I(t) + \xi D(t) - (\gamma+\alpha)W(t) \tag{27b}$$

$${}_0^{ABC}D_t^q D(t) = \pi I(t) - \left(\xi+\alpha_1\right)D(t). \tag{27c}$$

From (27) we have

$$F_i = \begin{bmatrix} (1-\zeta)\left(\bar{h}\right)S(t) + (1-\eta)\left(\bar{h}\right)V(t) \\ 0 \\ 0 \end{bmatrix}$$

Partial differentiation of F_i with respect to $I(t)$, $W(t)$ and $D(t)$ yields

$$F = \left((1-\zeta)S^0 + (1-\eta)V^0\right)\begin{bmatrix} \beta_1 & \beta_2 & \beta_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Similarly

$$V_i = \begin{bmatrix} (\mathcal{X})I(t) & 0 & 0 \\ -\phi I(t) & (\gamma+\alpha)W(t) & -\xi D(t) \\ \pi I(t) & 0 & \left(\xi+\alpha_1\right)D(t) \end{bmatrix}$$

Partial differentiation of V_i with respect to $I(t)$, $W(t)$ and $D(t)$ yields

$$V = \begin{bmatrix} (\delta+\mu+\rho+\sigma) & 0 & 0 \\ -\phi & (\gamma+\alpha) & -\xi \\ \pi & 0 & \left(\xi+\alpha_1\right) \end{bmatrix}$$

$$V^{-1} = \begin{bmatrix} \frac{1}{(\delta+\mu+\rho+\sigma)} & 0 & 0 \\ \frac{\phi(\xi+\alpha_1)+\pi\xi}{(\delta+\mu+\rho+\sigma)(\gamma+\alpha)(\xi+\alpha_1)} & \frac{1}{(\gamma+\alpha)} & \frac{\xi}{(\gamma+\alpha)(\xi+\alpha_1)} \\ \frac{\pi}{(\delta+\mu+\rho+\sigma)(\xi+\alpha_1)} & 0 & \frac{1}{(\xi+\alpha_1)} \end{bmatrix}$$

$$FV^{-1} = \left((1-\zeta)S^0 + (1-\eta)V^0\right)$$

$$\begin{bmatrix} \beta_1 & \beta_2 & \beta_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \frac{1}{\mathcal{X}} & 0 & 0 \\ \frac{\phi(\xi+\alpha_1)+\pi\xi}{\mathcal{X}\bar{\gamma}(\xi+\alpha_1)} & \frac{1}{\bar{\gamma}} & \frac{\xi}{\bar{\gamma}(\xi+\alpha_1)} \\ \frac{\pi}{\mathcal{X}(\xi+\alpha_1)} & 0 & \frac{1}{(\xi+\alpha_1)} \end{bmatrix}.$$

where $\bar{\gamma} = \gamma+\alpha$ and $\bar{\beta_2} = \beta_2\left(\phi\xi_{\alpha_1} + \pi\xi\right)$.

$$FV^{-1} = \left((1-\zeta)S^0 + (1-\eta)V^0\right)$$

$$\begin{bmatrix} \frac{\beta_1}{\mathcal{X}} + \frac{\bar{\beta_2}}{\mathcal{X}(\gamma+\alpha)\left(\xi_{\alpha_1}\right)} + \frac{\beta_3\pi}{\mathcal{X}\left(\xi_{\alpha_1}\right)} & \frac{\beta_2}{\bar{\gamma}} & \frac{\beta_2\xi}{\bar{\gamma}(\xi+\alpha_1)} + \frac{\beta_3}{(\xi+\alpha_1)} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The spectral radius for FV^{-1} gives the effective reproduction number of the waterborne disease model which is denoted by $\mathcal{R}_0^{eff}$ and after substituting the value of S^0 *and* V^0 from equation (20) we have

$$\mathcal{R}_0^{eff} = \psi\left(\frac{\mu(1-\zeta)+\omega(1-\eta)}{\mu(\mu+\omega+\varepsilon)}\right)\times\cdots \left[\frac{\beta_1\bar{\gamma}\left(\xi+\alpha_1\right)+\beta_2\left(\phi\left(\xi+\alpha_1\right)+\pi\xi\right)+\beta_3\pi\bar{\gamma}}{\mathcal{X}\bar{\gamma}\left(\xi+\alpha_1\right)}\right]. \tag{28}$$

Therefore, $\mathcal{R}_0^{eff}$, is the effective reproduction number (basic reproduction number of model system (3) with controls). When there is no intervention of any kind, we have that $\zeta = \omega = \sigma = \alpha = \alpha_1 = 0$, thus, the basic reproduction number of model system (3) will be given as

$$\mathcal{R}_0 = \psi\left[\frac{\beta_1\xi\gamma+\beta_2\xi(\phi+\pi)+\beta_3\pi\gamma}{\mu\xi\gamma(\delta+\mu+\rho)}\right]. \tag{29}$$

3.1. Analysis of $\mathcal{R}_0^{eff}$ with Single Control Strategy in Reducing Pathogenic Contamination

This section considers the individual control strategies using the effective reproduction number in system (28). If adequate supply and consumption of potable water is the only control strategy available, that is $\alpha \neq 0,\ \zeta = \omega = \sigma = \alpha_1 = 0$, then the basic reproduction number in the presence of adequate supply and consumption of potable water only is given as

$$\mathcal{R}_{0\alpha} = \psi\left[\frac{\beta_1\xi(\gamma+\alpha)+\beta_2\xi(\phi+\pi)+\beta_3\pi(\gamma+\alpha)}{\mu\xi(\delta+\mu+\rho)(\gamma+\alpha)}\right]. \tag{30}$$

If vaccination is the only control strategy available, that is $\omega \neq 0,\ \zeta = \alpha = \sigma = \alpha_1 = 0$, then the $\mathcal{R}_0$ in the presence of vaccination only is given as

$$\mathcal{R}_{0\omega} = \psi\left(\frac{\mu+\omega(1-\eta)}{\mu(\mu+\omega)}\right)\left[\frac{\beta_1\gamma\xi+\beta_2\xi(\phi+\pi)+\beta_3\pi\gamma}{\gamma\xi(\delta+\mu+\rho)}\right]. \tag{31}$$

If treatment is the only control strategy available, that is $\sigma \neq 0,\ \zeta = \alpha = \omega = \alpha_1 = 0$, then the basic reproduction number in the presence of treatment only is given as

$$\mathcal{R}_{0\sigma} = \psi\left[\frac{\beta_1\gamma\xi+\beta_2\xi(\phi+\pi)+\beta_3\pi\gamma}{\mu\gamma\xi(\delta+\mu+\rho+\sigma)}\right]. \tag{32}$$

If health education campaigns are the only control strategy available, that is $\zeta \neq 0,\ \sigma = \alpha = \omega = \alpha_1 = 0$, then the basic reproduction number in the presence of health education campaign only is given as

$$\mathcal{R}_{0\zeta} = \psi\left(\frac{(1-\zeta)}{\mu}\right)\left[\frac{\beta_1\gamma\xi+\beta_2\xi(\phi+\pi)+\beta_3\pi\gamma}{\gamma\xi(\delta+\mu+\rho)}\right]. \tag{33}$$

If sanitation is the only control strategy available, that is $\alpha_1 \neq 0,\ \zeta = \alpha = \omega = \sigma = 0$, then the basic reproduction number in the presence of sanitation only is given as

$$\mathcal{R}_{0\alpha_1} = \psi\left[\frac{\beta_1\gamma\xi_{\alpha_1}+\beta_2\left(\phi\xi_{\alpha_1}+\pi\xi\right)+\beta_3\pi\gamma}{\mu\gamma(\delta+\mu+\rho)\xi_{\alpha_1}}\right], \tag{34}$$

where $\xi_{\alpha_1} = \left(\xi+\alpha_1\right)$.

3.2. Analysis of $\mathcal{R}_0^{eff}$ with Two Control Strategies in Reducing Pathogenic Contamination

This section further considered the two control strategies by analysing system (28). If adequate supply and consumption of potable water with vaccination are the only available control, that is, $\alpha \neq 0, \omega \neq 0, \zeta = \alpha_1 = \sigma = 0$, then we have the $\mathcal{R}_0$ in the presence of adequate supply and consumption of potable water with vaccination as

$$\mathcal{R}_{0\alpha\omega} = \psi\left(\frac{\mu+\bar{\omega}}{\mu\left(\mu_\varepsilon+\omega\right)}\right)\left[\frac{\beta_1\xi\bar{\gamma}+\beta_2\xi(\phi+\pi)+\beta_3\pi\bar{\gamma}}{\xi_\delta\bar{\gamma}}\right]. \tag{35}$$

where $\bar{\omega} = \omega(1-\eta)$ and $\xi_\delta = \xi(\delta+\mu+\rho)$.

If adequate supply and consumption of potable water with treatment are the only available control, that is,$\alpha \neq 0, \sigma \neq 0,\ \zeta = \alpha_1 = \omega = 0$, then we have the basic reproduction number in the presence of adequate supply and consumption of potable water with treatment as

$$\mathcal{R}_{0\alpha\sigma} = \left(\frac{\psi}{\mu_\varepsilon\mu}\right)\left[\frac{\beta_1\xi\bar{\gamma}+\beta_2\xi(\phi+\pi)+\beta_3\pi\bar{\gamma}}{\xi\mathcal{X}\bar{\gamma}}\right]. \tag{36}$$

where $\mu_\varepsilon = (\mu+\varepsilon)$. If adequate supply and consumption of potable water with health education campaigns are the only available control, that is,$\alpha \neq 0, \zeta \neq 0, \sigma = \alpha_1 = \omega = 0$, then we have the basic reproduction number in the presence of adequate supply and consumption of potable water with health education campaign as

$$\mathcal{R}_{0\alpha\zeta} = \psi\left(\frac{(1-\zeta)}{\mu_\varepsilon}\right)\left[\frac{\beta_1\xi\bar{\gamma}+\beta_2\xi(\phi+\pi)+\beta_3\pi\bar{\gamma}}{\xi(\delta+\mu+\rho)\bar{\gamma}}\right]. \tag{37}$$

If adequate supply and consumption of potable water with sanitation are the only available control, that is,$\alpha \neq 0, \alpha_1 \neq 0, \sigma = \zeta = \omega = 0$, then we have the basic reproduction number in the presence of adequate supply and consumption of potable water with sanitation as

$$\mathcal{R}_{0\alpha\alpha_1} = \left(\frac{\psi}{\mu_\varepsilon}\right)\left[\frac{\beta_1\bar{\gamma}\xi_{\alpha_1}+\bar{\beta_2}+\beta_3\pi\bar{\gamma}}{(\delta+\mu+\rho)\bar{\gamma}\xi_{\alpha_1}}\right], \tag{38}$$

3.3. Analysis of $\mathcal{R}_0^{eff}$ with Three Control Strategies in Reducing Pathogenic Contamination

Again, we analyse system (28) by considering three control strategies (Intervention). If adequate supply and consumption of potable water, vaccination and treatment are the only available control, that is, $\alpha \neq 0, \omega \neq 0, \sigma \neq 0, \zeta = \alpha_1 = 0$, then we have the basic reproduction number in

$$\mathcal{R}_{0\alpha\omega\sigma\alpha_1} = \psi\left(\frac{\mu+\omega(1-\eta)}{\mu(\mu+\omega+\varepsilon)}\right)\left[\frac{\beta_1(\gamma+\alpha)(\xi+\alpha_1)+\beta_2(\phi(\xi+\alpha_1)+\pi\xi)+\beta_3\pi(\gamma+\alpha)}{(\delta+\mu+\rho+\sigma)(\gamma+\alpha)(\xi+\alpha_1)}\right]. \tag{43}$$

the presence of adequate supply and consumption of potable water, vaccination and treatment as

$$\mathcal{R}_{0\alpha\omega\sigma} = \psi\left(\frac{\mu+\bar{\omega}}{\mu(\mu_\varepsilon+\omega)}\right)\left[\frac{\beta_1\xi\bar{\gamma}+\beta_2\xi(\phi+\pi)+\beta_3\pi\bar{\gamma}}{\xi(\delta+\mu+\rho+\sigma)(\xi+\alpha_1)}\right]. \tag{39}$$

If adequate supply and consumption of potable water, vaccination and health education campaigns are the only available control, that is, $\alpha \neq 0, \omega \neq 0, \zeta \neq 0, \sigma = \alpha_1 = 0$, then we have the basic reproduction number in the presence of adequate supply and consumption of potable water, vaccination and health education campaign as

$$\mathcal{R}_{0\alpha\omega\zeta} = \psi\left(\frac{\mu(1-\zeta)+\bar{\omega}}{\mu(\mu_\varepsilon+\omega)}\right)\left[\frac{\beta_1\xi\bar{\gamma}+\beta_2\xi(\phi+\pi)+\beta_3\pi\bar{\gamma}}{\xi(\delta+\mu+\rho)\bar{\gamma}}\right]. \tag{40}$$

If adequate supply and consumption of potable water, vaccination and Sanitation are the only available control, that is, $\alpha \neq 0,\ \omega \neq 0,\ \alpha_1 \neq 0,\ \zeta = \sigma = 0$, then we have the basic reproduction number in the presence of adequate supply and consumption of potable water, vaccination and Sanitation as

$$\mathcal{R}_{0\alpha\omega\alpha_1} = \psi\left(\frac{\mu+\bar{\omega}}{\mu(\mu_\varepsilon+\omega)}\right)\left[\frac{\beta_1\bar{\gamma}\xi_{\alpha_1}+\bar{\beta}_2+\beta_3\pi\bar{\gamma}}{(\delta+\mu+\rho)\bar{\gamma}\xi_{\alpha_1}}\right]. \tag{41}$$

3.4. Analysis of $\mathcal{R}_0^{eff}$ with Four Control Strategies in Reducing Pathogenic Contamination

Finally, we analyse system (28) by combining four control strategies (Intervention). If adequate supply and consumption of potable water, vaccination, treatment, and health education campaigns are the only available control, that is, $\alpha \neq 0, \omega \neq 0,\ \sigma \neq 0,\ \zeta \neq 0,\ \alpha_1 = 0$, then we have the basic reproduction number in the presence of adequate supply and consumption of potable water, vaccination treatment and health education campaign as

$$\mathcal{R}_{0\alpha\omega\sigma\zeta} = \psi\left(\frac{\bar{\mu}+\bar{\omega}}{\mu(\mu_\varepsilon+\omega)}\right)\left[\frac{\beta_1\xi\bar{\gamma}+\beta_2\xi(\phi+\pi)+\beta_3\pi\bar{\gamma}}{\xi(\delta+\mu+\rho+\sigma)\bar{\gamma}}\right]. \tag{42}$$

where $\bar{\mu} = \mu(1-\zeta)$. If adequate supply and consumption of potable water, vaccination, treatment and sanitation are the only available control, that is, $\alpha \neq 0, \omega \neq 0, \sigma \neq 0,\ \alpha_1 \neq 0,\ \zeta = 0$, then we have the basic reproduction number in the presence of adequate supply and consumption of potable water, vaccination treatment and sanitation as

Theorem 3. *The disease-free equilibrium point E^0 of system (3) is locally asymptotically stable if $\mathcal{R}_0^{eff} < 1$ and unstable if $\mathcal{R}_0^{eff} > 1$.*

Proof. The Jacobian matrix associated with Model system (3) is obtained as (44).

$$J(E^0) = \begin{bmatrix} -G & 0 & m_{13} & 0 & 0 & m_{16} & m_{17} \\ \omega^q & \mu^{-q} & m_{23} & 0 & 0 & m_{26} & m_{27} \\ 0 & 0 & m_{33} & -H & 0 & m_{36} & m_{37} \\ 0 & 0 & \sigma^q & -E & 0 & 0 & 0 \\ \epsilon^q & 0 & \rho^q & \varphi^q & -\mu^q & 0 & 0 \\ 0 & 0 & \theta^q & 0 & 0 & -C & \xi^q \\ 0 & 0 & \pi^q & -E & 0 & 0 & -B \end{bmatrix} \tag{44}$$

where $m_{13} = \frac{\psi^q{\beta_1}^q(1-\zeta^q)}{\mu^q+\omega^q+\varepsilon^q}$, $m_{16} = \frac{\psi^q{\beta_2}^q(1-\zeta^q)}{\mu^q+\omega^q+\varepsilon^q}$, $m_{17} = \frac{\psi^q{\beta_3}^q(1-\zeta^q)}{\mu^q+\omega^q+\varepsilon^q}$, $m_{23} = \frac{\psi^q\omega^q(1-\eta^q)}{\mu^q(\mu^q+\omega^q+\varepsilon^q)}$ $m_{26} = \frac{\psi^q\omega^q{\beta_2}^q(1-\eta^q)}{\mu^q(\mu^q+\omega^q+\varepsilon^q)}$, $m_{33} = \frac{\psi^q{\beta_1}^q(\mu^q(1-\zeta^q))+\omega^q(1-\eta^q)}{\mu^q(\mu^q+\omega^q+\varepsilon^q)}$, $m_{36} = \frac{\psi^q{\beta_2}^q(\mu^q(1-\zeta^q))+\omega^q(1-\eta^q)}{\mu^q(\mu^q+\omega^q+\varepsilon^q)}$,$m_{37} = \frac{\psi^q{\beta_3}^q(\mu^q(1-\zeta^q))+\omega^q(1-\eta^q)}{\mu^q(\mu^q+\omega^q+\varepsilon^q)}$, $B = \xi^q + {\alpha_1}^q$,$C = \gamma^q + \alpha^q$, $E = \mu^q + \varphi^q$, $G = \mu^q + \omega^q + \varepsilon^q$, $H = (\delta^q + \mu^q + \rho^q + \sigma^q)$.

It follows that $\left|\arg\left(\lambda_i\right)\right| > \frac{q\pi}{2}$ $(i = 1, 2, \ldots, 7)$, where $0 < q < 1$. If we observe critically the form of the Jacobian matrix $J\left(E^0\right)$ above, we notice the three eigenvalues of $J\left(E^0\right)$ as $--\mu$, $-E$ and $-\mu$. The remaining eigenvalues are obtained from 4×4 submatrix given by

The characteristic determinant is given by $|P-\lambda I|$while the characteristic equation is given as $|P-\lambda I| = 0$ from which we obtain:

$$|P-\lambda I| = \begin{vmatrix} -G-\lambda & m_13 & m_16 & m_17 \\ 0 & m_33-H-\lambda & m_36 & m_37 \\ 0 & \phi^q & -C-\lambda & \xi^q \\ 0\pi^q & 0 & \lambda & \end{vmatrix} \tag{45}$$

The characteristic polynomial of the matrix P in terms of λ is given below:

$$P(\lambda) = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0 \tag{46}$$

Resolving the characteristic equation yields

$$(-G-\lambda)(-BCH - BC\lambda + BCm_{33} - BH\lambda - B\lambda^2 + B\lambda m_{33} + B\phi m_{36} - CH\lambda + C\pi M_{37} - C\lambda^2 + C\lambda m_{33} - H\lambda^2 + \pi\lambda m_3 7 + \pi\xi m_{36} - \lambda^3 + \lambda^2 m_{33} + \lambda\phi m_{36}).$$

The characteristic equation of the matrix

$$P(\lambda) = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0,$$

can be written as in (47), where the terms are defined as By the Routh Hurwitz criterion, as in Lima [48], it follows that all roots of the characteristic equation (46) has negative real parts if and only if $a_1 > 0,\ a_3 > 0,\ a_4 > 0$ and $a_1a_2a_3 > a_1^2a_4 + a_3^2$ with $\mathcal{R}_0^{eff} < 1$, it simply shows that the roots of $P(\lambda)$ has a negative real part and this proves the desired result. □

$$P(\lambda) = \lambda^4 + \left[B + C + H + G - m_{33}\right]\lambda^3 + \left[BC + BH + CH + BG + CG + GH - (B + C + G)m_{33} - \pi m_{37} - \phi m_{36}\right]\lambda^2 + \left[BCG + BGH + CGH - BGm_{33} - CGm_{33} - G\pi m_{37} - \phi G m_{36} + \left[1 - \mathcal{R}_0^{eff}\right]\right]\lambda + G\left[1 - \mathcal{R}_0^{eff}\right] = 0, \quad (47)$$

$$a_1 = B + C + H + G - m_{33}, a_2 = BC + BH + CH + BG + CG + GH - Bm_{33} - Cm_{33} - \pi^q m_{37} - \phi^q m_{36} - Gm_{33}$$
$$a_3 = BCG + BGH + CGH - BGm_{33} - CGm_{33} - G\pi m_{37} - \phi^q G m_{36} + \left[1 - \mathcal{R}_0^{eff}\right] a_4 = G\left[1 - \mathcal{R}_0^{eff}\right]. \quad (48)$$

$$\min \mathcal{J}\left(A_1, A_2, A_3, \mathcal{T}\right) = \int_0^{t_f} \left(A_1 I + A_2 W + A_3 D + \frac{1}{2}\left[C_1\tau_1^2(t) + C_2\tau_2^2(t) + C_3\tau_3^2(t) + C_4\tau_4^2(t) + C_5\tau_5^2(t)\right]\right)dt \quad (49a)$$

Subject to :

$$ {}_0^{ABC}D_t^q S(t) = \psi - \left(1 - \tau_1(t)\right)\left(\beta_1 I(t) + \beta_2 W(t) + \beta_3 D(t)\right) S(t) - \left(\mu + \tau_2(t) + \varepsilon\right) S(t) \quad (49b)$$
$$ {}_0^{ABC}D_t^q V(t) = \tau_2(t) S(t) - (1 - \eta)\left(\beta_1 I(t) + \beta_2 W(t) + \beta_3 D(t)\right) V(t) - \mu V(t) \quad (49c)$$
$$ {}_0^{ABC}D_t^q I(t) = \left(1 - \tau_1(t)\right)\bar{h} S(t) + (1 - \eta)\bar{h} V(t) - \left(\delta + \mu + \rho + \tau_3(t)\right) I(t) \quad (49d)$$
$$ {}_0^{ABC}D_t^q T(t) = \tau_3(t) I(t) - (\mu + \varphi) T(t) \quad (49e)$$
$$ {}_0^{ABC}D_t^q R(t) = \varphi T(t) + \rho I(t) + \varepsilon S(t) - \mu R(t) \quad (49f)$$
$$ {}_0^{ABC}D_t^q W(t) = \phi I(t) + \xi D(t) - \left(\gamma + \tau_4(t)\right) W(t) \quad (49g)$$
$$ {}_0^{ABC}D_t^q D(t) = \pi I(t) - \left(\xi + \tau_5(t)\right) D(t). \quad (49h)$$

4. Fractional Optimal Control of Waterborne Disease Model

Let's now explore the optimal control problems and conditions that can reduce the reproduction number in the model. A major concern is the affordability of these control interventions, especially since many communities have limited resources. Therefore, we need to focus on minimising the cost of these control strategies or interventions. Since optimal control theory has been successfully used in analysing problems such as in [49, 50, 51, 52]. We aim to minimise the cost of implementation of these control interventions (strategies) by making the following assumptions;

1. We assume that the control parameters $\zeta, \alpha, \omega, \sigma$and α_1 are measurable functions of time and formulate an appropriate optimal control which minimises the cost of implementing the model system (3).
2. We assume that $\tau_1(t) = \zeta,\ \tau_2(t) = \omega,\ \tau_3(t) = \sigma,\ \tau_4(t) = \alpha,\ \tau_5(t) = \alpha_1$. Here, the feasible control function of the above model system (3) on $\mathbb{R}_+^7$ is considered as

$$\Gamma = \left\{(\mathcal{T}) \in \left(\Pi^{\infty}\left(0,\ t_f\right)\right),\ 0 \le \mathcal{T} \le 1, \forall t \in \left[0,\ t_f\right]\right\}.$$

Hence, the fractional optimal control problem below is completely analysed (49).

With the following initial conditions

$$S(0) = S^0 \ge 0,\ V(0) = V^0 \ge 0,\ I(0) = I^0 \ge 0,$$
$$T(0) = T^0 \ge 0,\ R(0) = R^0 \ge 0,\ W(0) = W^0 \ge 0,$$
$$D(0) = D^0 \ge 0 \quad (50)$$

Thus, the optimal control:

$$\tau^*(t) = \left(\tau_1^*(t),\ \tau_2^*(t),\ \tau_3^*(t),\ \tau_4^*(t),\ \tau_5^*(t)\right) \in \Gamma$$

is required so that

$$\mathcal{J}\left(\tau^*\right) = \min_{\mathcal{T} \in \Gamma} \mathcal{J}(\mathcal{T}) \quad (51)$$

is reduced, where $\mathcal{T} = \left(\tau_1(t), \tau_2(t), \tau_3(t), \tau_4(t), \tau_5(t)\right)$. We observe that: $\frac{\partial \mathcal{R}_0^{eff}}{\partial \tau_1} < 0$, $\frac{\partial \mathcal{R}_0^{eff}}{\partial \tau_2} < 0$, $\frac{\partial \mathcal{R}_0^{eff}}{\partial \tau_3} < 0$, $\frac{\partial \mathcal{R}_0^{eff}}{\partial \tau_4} < 0$, and $\frac{\partial \mathcal{R}_0^{eff}}{\partial \tau_5} < 0$. Then, we can determine the optimal value of $\tau_1,\ \tau_2,\ \tau_3,\ \tau_4$ and τ_5 to make $\mathcal{R}_0^{eff} < 1$.

4.1. Existence of an Optimal Control of Waterborne Diseases

The existence of the optimal control pair for the state system (49) can be proved by showing the existence of the state variable and the existence of the objective functional [53] since the state variables describe the state of the dynamics system.

Theorem 4: Consider the control problem with system (49), there exist $\vec{\tau}^* = \left(\tau_1^*(t),\ \tau_2^*(t),\ \tau_3^*(t),\ \tau_4^*(t),\ \tau_5^*(t)\right) \in \Gamma$ such that

$$\mathcal{J}\left(\vec{\tau}^*\right) = \min_{\mathcal{T} \in \Gamma} \mathcal{J}(\mathcal{T}). \quad (52)$$

Proof. To use an existence result, there exists an optimal control which minimises $\mathcal{J}(\mathcal{T})$ if the following conditions are satisfied [53, 54, 55, 56]:

1. The set of controls and corresponding state variables F is non-empty.
2. The control set Γ must be closed and convex.
3. The right-hand side of the system is continuous, bounded above by a linear combination of the control and state variables and can be written as a linear function τ with coefficients defined by the time and state.

4. The integrand of the objective functional is convex on Γ and is bounded below by $L_1(\tau)^{\beta} - L_2$ with $L_1 > 0$ and $\beta > 1$.

To prove the first condition, the set of controls and the corresponding state variable F is non-empty, we use a simplified version of an existence result by [57]. Let $Y_i' = F_{X_i}(t; Y_S, Y_V, Y_I, Y_T, Y_R, Y_W, Y_D)$ where

$$(Y_S, Y_V, Y_I, Y_T, Y_R, Y_W, Y_D) = \big(\mathrm{S}(t), \mathrm{V}(t), \mathrm{I}(t), \mathrm{T}(t), \mathrm{R}(t), \mathrm{W}(t), \mathrm{D}(t)\big)$$

and $Y_S, Y_V, Y_I, Y_T, Y_R, Y_W$ and Y_D form the right hand side of the system of equations (59). Let $\tau_1, \tau_2, \tau_3, \tau_4$ and τ_5 for some constants and since all parameters and coefficients and $Y_S, Y_V, Y_I, Y_T, Y_R, Y_W$ and Y_D are continuous, then $F_S, F_V, F_I, F_T, F_R, F_W$ and F_D are also continuous. Additionally, the partial derivatives $\frac{\partial F_S}{\partial Y_S}$, $\frac{\partial F_V}{\partial Y_V}$, $\frac{\partial F_I}{\partial Y_I}$, $\frac{\partial F_T}{\partial Y_T}$, $\frac{\partial F_R}{\partial Y_R}$, $\frac{\partial F_W}{\partial Y_W}$, $\frac{\partial F_D}{\partial Y_D}$ are all continuous. Therefore, there exists a unique solution $(\mathrm{S}(t), \mathrm{V}(t), \mathrm{I}(t), \mathrm{T}(t), \mathrm{R}(t), \mathrm{W}(t), \mathrm{D}(t))$ that satisfies the initial conditions. Therefore, the set of controls and the corresponding state variables is non-empty and Condition 1 is satisfied.

To prove the second condition, that is, the control set of Γ must be closed and convex, we define that

$$\mathcal{J}(\mathcal{T}) = \int_0^{t_f} \Big(A_1 I + A_2 W + A_3 D + \frac{1}{2}\big[C_1\tau_1^2(t) + \cdots C_2\tau_2^2(t) + C_3\tau_3^2(t) + C_4\tau_4^2(t) + C_5\tau_5^2(t)\big]\Big) dt \tag{53}$$

is convex in Γ.

Take any two controls $\tau_1,\ \tau_2 \in \Gamma$and $\Lambda \in [0,\ 1]$, then $0 \leq \Lambda\tau_1 + (1-\Lambda)\tau_2$. Additionally, we observe that $\Lambda\tau_1 \leq \Lambda$and $(1-\Lambda)\tau_2 \leq (1-\Lambda)$, then $\Lambda\tau_1 + (1-\Lambda)\tau_2 \leq \Lambda + (1-\Lambda) = 1$. Hence, $0 \leq \Lambda\tau_1 + (1-\Lambda)\tau_2 \leq \Lambda + (1-\Lambda) \leq 1$, for all $\tau_1,\ \tau_2 \in \Gamma$ and $\Lambda \in [0,\ 1]$. We have the control space $\Gamma = \frac{(\tau_1,\ \tau_2,\ \tau_3,\ \tau_4,\ \tau_5)}{(\tau_1,\ \tau_2,\ \tau_3,\ \tau_4,\ \tau_5)}$ is measurable, $0 \leq \tau_{1min} \leq \tau_1(t) \leq \tau_{1max} \leq 1,\ 0 \leq \tau_{2min} \leq \tau_2(t) \leq \tau_{2max} \leq 1,\ 0 \leq \tau_{3min} \leq \tau_3(t) \leq \tau_{3max} \leq 1,\ 0 \leq \tau_{4min} \leq \tau_4(t) \leq \tau_{4max} \leq 1,\ 0 \leq \tau_{5min} \leq \tau_5(t) \leq \tau_{5max} \leq 1,\ t \in \big[0,\ t_f\big]$ is convex and closed by definition.

We need to show that the entire right side of system (49) is continuous in order to prove the third condition. It is bounded above by the sum of bounded control and state, and it can be expressed as a linear function of $\tau_1,\ \tau_2,\ \tau_3,\ \tau_4$ *and* τ_5, with coefficients that depend on time and state from the fractional derivative system in (49).

$$ {}_0^{ABC}D_t^q N_g(t) = \psi - \mu N_g(t) \tag{54a}$$

$$\lim_{t\to\infty} Sup\ N_g(t) \leq \frac{\psi}{\mu} \tag{54b}$$

Therefore, all solutions of the system (49) are bounded. So there exist positive constants $X_S, X_V, X_I, X_T, X_R, X_W$ and X_D for all $t \in \big[0, t_f\big] : S(t) \leq X_S, V(t) \leq X_V, I(t) \leq X_I, T(t) \leq X_T, R(t) \leq X_R, W(t) \leq X_W$ and $D(t) \leq X_D$

$$F_S = S(t) \leq \psi - \big(\tau_1 + \tau_2\big) X_S \tag{55a}$$

$$F_V = V(t) \leq \omega S(t) \tag{55b}$$

$$F_I = I(t) \leq \frac{\big(\big(1-\tau_1\big) X_S + (1-\eta)\big) AI(t)}{BC} - \tau_2 X_I \tag{55c}$$

$$F_T = T(t) \leq \sigma I(t) \tag{55d}$$

$$F_R = R(t) \leq \varphi T(t) + \rho I(t) + \varepsilon S(t) \tag{55e}$$

$$F_W = W(t) \leq \phi I(t) + \xi D(t) - \tau_4 X_W \tag{55f}$$

$$F_D = D(t) \leq \pi I(t) - \tau_5 X_D. \tag{55g}$$

If we let $\mathcal{F} = [S(t),\ V(t),\ I(t),\ T(t),\ R(t),\ W(t),\ D(t)]$, then we can rewrite system (49) in matrix form as

$$F(t; \mathcal{F}) \leq Z + B_1 X(t) - B_2 \Gamma(t) \tag{56}$$

where

$$F(t; \mathcal{F}) = \big[F_S,\ F_V,\ F_I,\ F_T,\ F_R,\ F_W,\ F_D\big]^T \tag{57a}$$

$$Z = [\psi,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0]^T \tag{57b}$$

$$X(t) = \mathcal{F}^T \tag{57c}$$

$$\Gamma(t) = \big[\tau_1,\ \tau_2,\ \tau_3,\ \tau_4,\ \tau_5\big]^T, \tag{57d}$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \omega & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma & 0 & 0 & 0 & 0 \\ \epsilon & 0 & \rho & \varphi & 0 & 0 & 0 \\ 0 & 0 & \phi & 0 & 0 & 0 & \xi \\ 0 & 0 & \pi & 0 & 0 & 0 & 0 \end{bmatrix} \tag{57e}$$

$$B_2 = \begin{bmatrix} b_{211} & -S & 0 & 0 & 0 \\ b_{221} & S & 0 & 0 & 0 \\ b_{231} & 0 & -I & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -W & 0 \\ 0 & 0 & 0 & 0 & -D \end{bmatrix} \tag{57f}$$

where $b_{211} = -AIS/BC$, $b_{221} = -(1-\eta)AIV/BC$ and $b_{231} = (S + (1-\eta)V)AI/BC$. It gives a linear function of the control Vector and state variable vector. Therefore, we can write

$$\begin{aligned} F(t; \mathcal{F}) &\leq \|Z\| + \big\|B_1\big\|\,\|X(t)\| + \big\|B_2\big\|\,\|\Gamma(t)\| \\ &\leq \mathfrak{Y}_1 + \mathfrak{Y}_2\,(\|\mathrm{X}(t)\| + \|\Gamma(t)\|) \end{aligned}$$

where $\mathfrak{Y}_1 \leq \|\psi\|$ and $\mathfrak{Y}_2 = \max\big(\big\|B_1\big\|,\ \big\|B_2\big\|\big)$. Hence, we see that the right-hand side is bounded by a sum of state and control vectors.

The prove of the fourth condition, that is, the integrand in the objective functional $A_1 I + A_2 W + A_3 D + \frac{1}{2}\big[C_1\tau_1^2(t) + C_2\tau_2^2(t) + C_3\tau_3^2(t) + C_4\tau_4^2(t) + C_5\tau_5^2(t)\big]$ is convex on Γ. we, then show that there exist constants $\Phi_1,\ \Phi_2,\ \Phi_3,\ \Phi_4,\ \Phi_5,\ \Phi_6 > 0$, and Φ such that $A_1 I + A_2 W + A_3 D + \frac{1}{2}\big[C_1\tau_1^2(t) + C_2\tau_2^2(t) + C_3\tau_3^2(t) + C_4\tau_4^2(t) + C_5\tau_5^2(t)\big]$ satisfies $A_1 I + A_2 W + A_3 D + \frac{1}{2}\big[C_1\tau_1^2(t) + C_2\tau_2^2(t) + C_3\tau_3^2(t) + C_4\tau_4^2(t) + C_5\tau_5^2(t)\big] \geq \Phi_1 + \Phi_2\big|\tau_1\big|^{\Phi} + \Phi_3\big|\tau_2\big|^{\Phi} + \Phi_4\big|\tau_3\big|^{\Phi} + \Phi_5\big|\tau_4\big|^{\Phi} + \Phi_6\big|\tau_5\big|^{\Phi}$.

For the state variables to be bounded, we let $\Phi_1 = \mathrm{Inf}_{t\in[0,\,t_f]}\left(A_1 I + A_2 W + A_3 D\right)$, $\Phi_2 = \frac{C_1}{2}$, $\Phi_3 = \frac{C_2}{2}$, $\Phi_4 = \frac{C_3}{2}$, $\Phi_5 = \frac{C_4}{2}$, $\Phi_6 = \frac{C_5}{2}$ and Φ= 2, then it follows that: $A_1 I + A_2 W + A_3 D + \frac{1}{2}\left[C_1\tau_1^2(t) + C_2\tau_2^2(t) + C_3\tau_3^2(t) + C_4\tau_4^2(t) + C_5\tau_5^2(t)\right] \geq \Phi_1 + \Phi_2\left|\tau_1\right|^{\Phi} + \Phi_3\left|\tau_2\right|^{\Phi} + \Phi_4\left|\tau_3\right|^{\Phi} + \Phi_5\left|\tau_4\right|^{\Phi} + \Phi_6\left|\tau_5\right|^{\Phi}$. Then, from [53], we conclude that there exists an optimal control. □

4.2. Optimal Control Characterisation

The optimality requirements are obtained by applying Pontryagin's maximal principle [58, 59, 60, 61], where the adjoint (co-state) functions connect the state equation to the cost Operation or Function $\mathcal{J}$. The method permits us to put across the control in terms of the state and adjoint functions. Furthermore, Pontryagin's maximum principle interprets the constraint maximisation problem (49) and system (50) as another maximisation problem (Hamiltonian) with respect to the controls. Thus, we have the Hamiltonian denoted by (58). where f_i is the right-hand side of the differential equation of the state variable by applying where $\lambda_S, \lambda_V, \lambda_I, \lambda_T, \lambda_R, \lambda_W$ and λ_D denote the adjoint variable associated with their corresponding state. Observe that the integrand in system (49) represents the first part of the Hamiltonian H, while the remaining parts are generated by multiplying each of the right-hand side of sub equations of system (49) by the adjoint functions λ_S, λ_V, λ_I, λ_T, λ_R, λ_W and λ_D respectively. Therefore, Pontryagin's maximum principle is guided by the following optimality system.
From (59), one can derive

$$
\begin{aligned}
&{}_0^{ABC}D_t^q\lambda_S = -\frac{\partial H}{\partial S},\ {}_0^{ABC}D_t^q\lambda_V = -\frac{\partial H}{\partial V},\\
&{}_0^{ABC}D_t^q\lambda_I = -\frac{\partial H}{\partial I},\ {}_0^{ABC}D_t^q\lambda_T = -\frac{\partial H}{\partial T},\\
&{}_0^{ABC}D_t^q\lambda_R = -\frac{\partial H}{\partial R},\ {}_0^{ABC}D_t^q\lambda_W = -\frac{\partial H}{\partial W},\\
&{}_0^{ABC}D_t^q\lambda_D = -\frac{\partial H}{\partial D}.
\end{aligned}
\tag{60}
$$

Moreover,

$$
\frac{\partial H}{\partial \tau_1} = 0,\ \frac{\partial H}{\partial \tau_2} = 0, \frac{\partial H}{\partial \tau_3} = 0,\ \frac{\partial H}{\partial \tau_4} = 0,\ \frac{\partial H}{\partial \tau_5} = 0\,.
\tag{61}
$$

Additionally, there is necessary condition such that λ_S, λ_V, λ_I, λ_T, λ_R, λ_W and λ_D are zero and system (60) provides the relevant state conditions. The fractional order Caputo problem in system (60) yields distinct sets of differential equations for the previously defined Hamiltonian, state variables $(S(t), V(t), I(t), T(t), R(t), W(t), D(t))$, controls$\left(\tau_1, \tau_2, \tau_3, \tau_4, \tau_5\right)$, and the Lagrangian.

Theorem 4. ***Theorem 5:*** *Given the optimal control pair* $\tau^*(t) = \left(\tau_1^*(t), \tau_2^*(t), \tau_3^*(t), \tau_4^*(t), \tau_5^*(t)\right) \in \Gamma$ *and the state solution* $Y^*(t) = S^*(t), V^*(t), I^*(t), T^*(t), R^*(t), W^*(t)$ *and* $D^*(t)$ *of the corresponding state system* (50) *that minimises* $\mathcal{J}\left(\tau_1, \tau_2, \tau_3, \tau_4, \tau_5\right)$ *in* (59)*, there exist adjoint variables* $\mathrm{T(t)} = \lambda_S(t), \lambda_V(t), \lambda_I(t), \lambda_T(t), \lambda_R(t), \lambda_W(t)$ *and* $\lambda_D(t)$ *satisfying system* (59)*. Differentiating system* (59) *with respect to* $S, V, I, T, R, W, D, \tau_1, \tau_2, \tau_3, \tau_4$ *and* τ_5 *we obtain* (62)*:*

The optimal equations were given by (63)

$$
-\frac{\partial H}{\partial \tau_1} = -C_1\tau_1 - \lambda_S\left(\left(\bar{h}\right)S(t)\right) + \lambda_I\left(\left(\bar{h}\right)S(t)\right) = 0 \quad \text{at } \tau_1^* \tag{63a}
$$

$$
-\frac{\partial H}{\partial \tau_2} = -C_2\tau_2 + \lambda_S S(t) - \lambda_V S(t) = 0 \text{ at } \tau_2^* \tag{63b}
$$

$$
-\frac{\partial H}{\partial \tau_3} = -C_3\tau_3 + \lambda_I I(t) - \lambda_T I(t) = 0 \text{ at } \tau_3^* \tag{63c}
$$

$$
-\frac{\partial H}{\partial \tau_4} = -C_4\tau_4 + \lambda_W W(t) = 0 \text{ at } \tau_4^* \tag{63d}
$$

$$
-\frac{\partial H}{\partial \tau_5} = -C_5\tau_5 + \lambda_D D(t) = 0 \text{ at } \tau_5^*. \tag{63e}
$$

Therefore

$$
\tau_1^* = \frac{\left(\lambda_I - \lambda_s\right)}{C_1}\left(\beta_1 I(t) + \beta_2 W(t) + \beta_3 D(t)\right)S(t) \tag{64a}
$$

$$
\tau_2^* = \frac{\left(\lambda_S - \lambda_V\right)S(t)}{C_2} \tag{64b}
$$

$$
\tau_3^* = \frac{\left(\lambda_I - \lambda_T\right)I(t)}{C_3} \tag{64c}
$$

$$
\tau_4^* = \frac{\lambda_W W(t)}{C_4} \tag{64d}
$$

$$
\tau_5^* = \frac{\lambda_D D(t)}{C_5}, \tag{64e}
$$

with the transversality conditions at time t_f : $\lambda_s\left(t_f\right) = 0$, $\lambda_V\left(t_f\right) = 0$, $\lambda_I\left(t_f\right) = -1$, $\lambda_T\left(t_f\right) = 0$, $\lambda_R\left(t_f\right) = 0$, $\lambda_W\left(t_f\right) = -1$, $\lambda_D\left(t_f\right) = -1$.Furthermore, for $t \in \left[0, t_f\right]$ and from system (64), the optimal controls $\tau_1^*(t), \tau_2^*(t), \tau_3^*(t), \tau_4^*(t)$ *and* $\tau_5^*(t)$ are given by (65)

$$
\tau_1^*(t) = \min\left\{1, \max\left\{0, \frac{\left(\lambda_I - \lambda_s\right)}{C_1}\left(h_1\right)S(t)\right\}\right\} \tag{65a}
$$

$$
\tau_2^*(t) = \min\left\{1, \max\left\{0, \frac{\left(\lambda_S - \lambda_V\right)S(t)}{C_2}\right\}\right\} \tag{65b}
$$

$$
\tau_3^*(t) = \min\left\{1, \max\left\{0, \frac{\left(\lambda_I - \lambda_T\right)I(t)}{C_3}\right\}\right\} \tag{65c}
$$

$$
\tau_4^*(t) = \min\left\{1, \max\left\{0, \frac{\lambda_W W(t)}{C_4}\right\}\right\} \tag{65d}
$$

$$
\tau_4^*(t) = \min\left\{1, \max\left\{0, \frac{\lambda_D D(t)}{C_5}\right\}\right\}. \tag{65e}
$$

Owing to the pair of boundedness of the state and adjoint functions in Γ of the controls and the resulting Lipschitz structure of ODEs, it is easy to obtain the uniqueness of the

$$H\left(Y,\tau,\mathrm{T}\right)=A_1 I+A_2 W+A_3 D+\frac{1}{2}\left[\sum_{\ell=1}^{5} C_\ell \tau_\ell^2\left(t\right)\right]+\sum_{i=S}^{D}\lambda_i f_i, \tag{58}$$

$$\begin{aligned}H(Y,\tau,\mathrm{T}) = {} & A_1 I+A_2 W+A_3 D+\frac{1}{2}\left[h_1\right]+\lambda_S\left(\psi-\left(1-\tau_1\left(t\right)\right)\left(\bar{h}\right)S\left(t\right)-\left(\mu+\tau_2\left(t\right)+\varepsilon\right)S\left(t\right)\right)+\lambda_V\left(\tau_2\left(t\right)S\left(t\right)-\right.\\ & \left.(1-\eta)\left(\bar{h}\right)V\left(t\right)-\mu V\left(t\right)\right)+\lambda_I\left(\left(1-\tau_1\left(t\right)\right)\left(\bar{h}\right)S\left(t\right)+(1-\eta)\left(\bar{h}\right)V\left(t\right)-\left(\delta+\mu+\rho+\tau_3\left(t\right)\right)I\left(t\right)\right)+\\ & \lambda_T\left(\tau_3\left(t\right)I\left(t\right)-(\mu+\varphi)T\left(t\right)\right)+\lambda_R\left(\varphi T\left(t\right)+\rho I\left(t\right)+\varepsilon S\left(t\right)-\mu R\left(t\right)\right)+\lambda_W\left(\phi I\left(t\right)+\xi D\left(t\right)-\left(\gamma+\tau_4\left(t\right)\right)W\left(t\right)\right)+\\ & \lambda_D\left(\pi I\left(t\right)-\left(\xi+\tau_5\left(t\right)\right)D\left(t\right)\right).\end{aligned} \tag{59}$$

where $h_1=C_1\tau_1^2\left(t\right)+C_2\tau_2^2\left(t\right)+C_3\tau_3^2\left(t\right)+C_4\tau_4^2\left(t\right)+C_5\tau_5^2\left(t\right)$,

$$\begin{aligned}
{}_0^{ABC}D_t^q\lambda_S &= \lambda_S\left(\left(1-\tau_1\left(t\right)\right)\left(\beta_1{}^q I\left(t\right)+\beta_2{}^q W\left(t\right)+\beta_3{}^q D\left(t\right)\right)+\left(\mu^q+\tau_2\left(t\right)+\varepsilon^q\right)\right)-\tau_2\left(t\right)\lambda_V-\\
&\quad \lambda_I\left(\left(1-\tau_1\left(t\right)\right)\left(\beta_1{}^q I\left(t\right)+\beta_2{}^q W\left(t\right)+\beta_3{}^q D\left(t\right)\right)\right)-\varepsilon^q\lambda_R && \text{(62a)}\\
{}_0^{ABC}D_t^q\lambda_V &= \lambda_V\left(\left(1-\eta^q\right)\left(\beta_1{}^q I\left(t\right)+\beta_2{}^q W\left(t\right)+\beta_3{}^q D\left(t\right)\right)+\mu^q\right)-\lambda_I\left(\left(1-\eta^q\right)\left(\beta_1{}^q I\left(t\right)+\beta_2{}^q W\left(t\right)+\beta_3{}^q D\left(t\right)\right)\right) && \text{(62b)}\\
{}_0^{ABC}D_t^q\lambda_I &= -A_1+\lambda_S\left(1-\tau_1\left(t\right)\right)\beta_1{}^q S\left(t\right)+\lambda_V\left(1-\eta^q\right)\beta_1{}^q V\left(t\right)-\lambda_I\left(\left(1-\tau_1\left(t\right)\right)\beta_1{}^q S\left(t\right)+\left(1-\eta^q\right)\beta_1{}^q V\left(t\right)\right)\\
&\quad +\left(\delta^q+\mu^q+\rho^q+\tau_3\left(t\right)\right)\lambda_I-\tau_3\left(t\right)\lambda_T-\rho\lambda_R-\phi\lambda_W-\pi\lambda_D && \text{(62c)}\\
{}_0^{ABC}D_t^q\lambda_T &= \left(\mu^q+\varphi^q\right)\lambda_T-\varphi^q\lambda_R && \text{(62d)}\\
{}_0^{ABC}D_t^q\lambda_R &= \mu^q\lambda_R && \text{(62e)}\\
{}_0^{ABC}D_t^q\lambda_W &= -A_2+\lambda_S\left(1-\tau_1\left(t\right)\right)\beta_2{}^q S\left(t\right)+\lambda_V\left(1-\eta^q\right)\beta_2{}^q V\left(t\right)-\lambda_I\left(\left(1-\tau_1\left(t\right)\right)\beta_2{}^q S\left(t\right)+\left(1-\eta^q\right)\beta_2{}^q V\left(t\right)\right)+\\
&\quad \left(\gamma^q+\tau_4\left(t\right)\right)\lambda_W && \text{(62f)}\\
{}_0^{ABC}D_t^q\lambda_D &= -A_3+\lambda_S\left(1-\tau_1\left(t\right)\right)\beta_3{}^q S\left(t\right)+\lambda_V\left(1-\eta^q\right)\beta_3{}^q V\left(t\right)-\lambda_I\left(\left(1-\tau_1\left(t\right)\right)\beta_3{}^q S\left(t\right)+\left(1-\eta^q\right)\beta_3{}^q V\left(t\right)\right)+\\
&\quad \left(\xi^q+\tau_5\left(t\right)\right)\lambda_D-\xi^q\lambda_W. && \text{(62g)}
\end{aligned}$$

optimal control pair which follows the optimality system as in [62].

4.3. Greedy Algorithm for Optimal Control of Waterborne Diseases

We present a brief context to the linear approximation theory of parameter problems utilising a sample Greedy Algorithms I and II. The Greedy Algorithm (GA) for an optimisation problem consistently selects the option that appears most favourable now and integrates it into the current sub-solutions. While the GAs do not guarantee an optimal solution in every case, they represent the simplest and most efficient available approach when successful. The GA operates in a step-by-step fashion. Initially, the set of chosen control interventions remains empty. Subsequently, at each step, we evaluate the addition of the best remaining untried control intervention strategies selected by the function to this set. To construct the solution in an optimal way, Algorithm I consists of four (4) functions which includes:

- The selection function tells which of the waterborne disease control strategies is the most promising, that is, the best-case scenario.
- A function that checks if a chosen set of control strategies provides a solution to waterborne diseases in the population or not (solution Function).
- A function that checks if the selected control strategies are feasible, that is, if it reduces the population of the infected individuals in the population and pathogen population in water reservoirs and dumpsites (Feasibility Function).
- An objective function that does not appear explicitly gives value of the solution.

In general, the goal is to approximate a compact set of controls K. This involves the selection of best control case scenario to be selected, and included in the set of controls after which it will be tried. The Algorithm I read as follows;

5. Numerical Simulation

In this section, we numerically solve the optimality system, and the results are also presented. In this model, there are initial conditions for the state variables and terminal conditions for the adjoints. In other words, the optimality system is a two-point boundary – value problem with separate boundary conditions at time $t=0$ and $t=t_f$ and our aim is to solve this problem for value $t_f=10$ $(months)$. Model system (3) is numerically simulated using a set of parameter values from Table 3. We conduct the numerical analysis using MATLAB, with the primary objective of applying the fractional derivative numerical approximation [63] to solve

Algorithm 1: Network Greedy Control

```
Input: Number of individuals N, time-period t,
Output: Classification_Results: Solution:", "result"
Begin: procedure()
  for t ← t to N do
    for i ← 1 to N do
      Def: greedy(control_strategies):
        'Control_strategies' is the set of Control Intervention
        Strategies
        Initialise the solution set 'S' as an empty set
        S = set()
        Function to solve waterborne disease problem using a greedy
        approach
        Def: greedy(control_strategies):
        'Control_strategies' is the set of Control Intervention
        Strategies
        Loop until the control strategies set is not empty and a
        solution is not found  while (Crypt_round = 1) do
          void setup ()
            while control_strategies not solution (S) do
              Select a strategy 'X' using the select function()
                X = select_strategy(control_strategies)
                control_strategies.remove(X);        /* Remove the
                selected strategy from the set of control
                strategies */
              Check if adding 'X' to the solution 'S' is feasible
                if is_feasible(S.union({X})): then
                |   S.add(X)
              end
              Add 'X' to the solution set 'S'
                Check if a solution is found
                if solution(S) then
                return # Return the solution set 'S';
              else:
                return "There is no solution ; /* Return a message
                indicating no solution found */
              Function to select the next strategy 'X'
                Def: select_strategy(control_strategies):
                Logic to select the strategy 'X' based on certain
                criteria
                Replace this with IoT_API_selection logic
                return  control_strategies. Pop() ;    /* # selects
                and pops the last strategy Function to check if
                the current solution 'S' is a valid solution */
              Def: solution(S):
                Logic to check if 'S' is a valid solution
                Replace this with IoT_API solution validation
                logic
                return len(S) > 0 ;      /* # For example, here it
                checks if the solution set is not empty */
              Function to check the feasibility of adding a strategy
                'X' to the current solution 'S'
                Def: is_feasible (S):
                Logic to check the feasibility of adding a strategy
                'X' to the solution 'S'
                Replace this with your feasibility check logic
                return True ;  /* # For example, here it assumes
                all strategies are feasible */
              {
                Until  round = Ran ( );
                ;                                  /* Example usage */
              control_interventions = { } #
                result = greedy(control_interventions)
                Return  ("Solution:", result)
            end
        end
    end
end
return Input
```

Algorithm 2: Control Greedy Algorithm

```
Input: Number of individuals N, time-period t
Output:  Optimal solution S
Begin procedure ()
for t ← t to K do
    for i ← 1 to C do
        Function: Function greedy(K:set) : set{
          ; /* // K is the set of Control Intervention Strategies */
        S← Φ ;          /* // construct the solution in the set S */
        while K ≠ Φ and not solution (S) do
            }
              X ← select (C)
              C ← C −{X}
              if feasible S ∪ {X} then
              S ← S ∪ {X};
            {
              }
        end
        while (Random_round = t do
            if solution, (S) then
              |   return S
            else
              |   return "there are no solution"
            end
        end
    end
end
return
```

a practical scenario. We set up the following:

$$ {}_{0}^{ABC}D_{t}^{q}S = f_1(t,\ S,\ V,\ I,\ T,\ R,\ W,\ D) \tag{66a} $$

$$ {}_{0}^{ABC}D_{t}^{q}V = f_2(t,\ S,\ V,\ I,\ T,\ R,\ W,\ D) \tag{66b} $$

$$ {}_{0}^{ABC}D_{t}^{q}I = f_3(t,\ S,\ V,\ I,\ T,\ R,\ W,\ D) \tag{66c} $$

$$ {}_{0}^{ABC}D_{t}^{q}T = f_4(t,\ S,\ V,\ I,\ T,\ R,\ W,\ D) \tag{66d} $$

$$ {}_{0}^{ABC}D_{t}^{q}R = f_5(t,\ S,\ V,\ I,\ T,\ R,\ W,\ D) \tag{66e} $$

$$ {}_{0}^{ABC}D_{t}^{q}W = f_6(t,\ S,\ V,\ I,\ T,\ R,\ W,\ D) \tag{66f} $$

$$ {}_{0}^{ABC}D_{t}^{q}D = f_7(t,\ S,\ V,\ I,\ T,\ R,\ W,\ D) \tag{66g} $$

such that $0 < \alpha < 1$ and $t > 1$. Suppose that the interval of the solution to the problem is $[0, n]$, the interval $[0, n]$ is divided into N sub-intervals, each with length $h = \frac{n}{N}$. The nodes are $t_i = ih$ where $i = 0, 1, 2, \cdots, N$ and $t_{i+1} = t_{i+h}$. The generalised formula is shown in Appendix A.

The goal of the optimal control infrastructure is to reduce the number of infected populations while using different combinations of control strategies in the model. The initial condition is $\left(S^0,\ V^0,\ I^0,\ T^0,\ R^0,\ W^0,\ D^0\right) =$ (2000, 4025, 1700, 288, 0, 1000, 1750). Since most of the parameter values were not readily available, we used data from the literature while the missing data were optimally estimated.

The effective reproduction number of the model was represented differently when considering controls versus when no control was considered. This is now given as With $(\zeta, \sigma, \omega, \alpha, \alpha_1) = (0.95, 0.6, 0.007, 0.5, 0.635)$, $\mathcal{R}_0^{eff} = 0.3592$.

5.1. Results and Discussions

In Figure 3, we show the variation in the reproduction number of pathogens when susceptible and vaccinated individuals contact the pathogens. The result from the figure showed that $\mathcal{R}_{0\alpha} < \mathcal{R}_{0\zeta} < \mathcal{R}_{0\omega} < \mathcal{R}_{0\sigma} < \mathcal{R}_{0\alpha_1} < \mathcal{R}_0$. Also, we see that $\mathcal{R}_0$ is the worst-case scenario, which occurs when there is no control strategy for waterborne diseases. The basic reproduction number $\mathcal{R}_0$ when there is no control grows exponentially with an increase in the disease transmission (contact) rate. Such an increase in $\mathcal{R}_0$ above unity implies a high spreading rate of waterborne diseases in the population. The best-case scenario for the unique control is $\mathcal{R}_{0\alpha}$. Here, an adequate potable water supply is the only

Table 3: Parameter values for the Waterborne Diseases Model

Parameter	Value	Source
ψ	10	[64]
β_1	0.042	[65, 66]
β_2	0.049	[65, 66, 67]
β_3	0.04	[65, 66, 67]
η	0.95	Estimated
ϕ	0.89	Estimated
ω	0.007	Estimated
μ	0.0005367	[68, 69]
φ	0.003	[70]
σ	0.6	Estimated
ε	0.5	Estimated
α	0.5	Estimated
ρ	0.98	[71]
ζ	0.95	[70]
γ	0.333	[72]
δ	0.0060	[72]
α_1	0.635	[67, 73]
ξ	0.32	[74]
π	0.05	Estimated

control intervention considered in the population. It can be noticed that the reproduction number of pathogens with a potable water supply strategy is minimal, which means that the disease can be eradicated from the population.

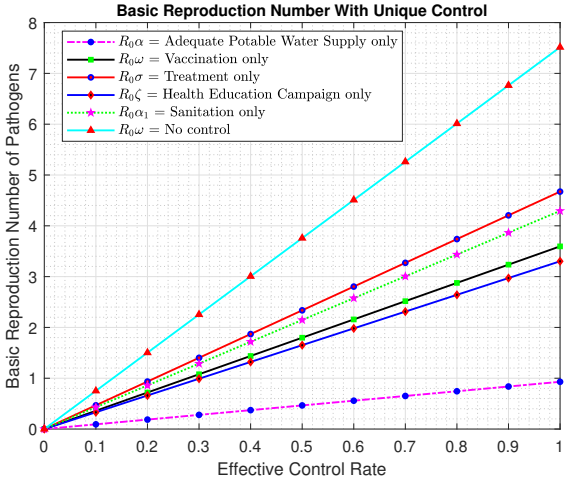

Figure 3. Variations in the reproduction number of pathogens when single control strategy is considered with respect to effective contact rate.

5.2. Effect of Control Strategies on Reproductive Number of Pathogenic Concentrations in Water

In Figure 4, we represent the variation in the effective reproduction number of pathogens with respect to individual control intervention strategy. It showed that when $\zeta = \omega = \sigma = \alpha = \alpha_1 = 1$, $\mathcal{R}_{0\zeta} < \mathcal{R}_{0\omega} < \mathcal{R}_{0\alpha} < \mathcal{R}_{0\sigma} < \mathcal{R}_{0\alpha_1}$. We observe that $\mathcal{R}_{0\alpha_1}$ is the worst-case scenario and this occurs when sanitation is the only available control intervention strategy. This implies that as the individual control strategy is increased, the rate at which the infection is eradicated in the population is minimal with sanitation.

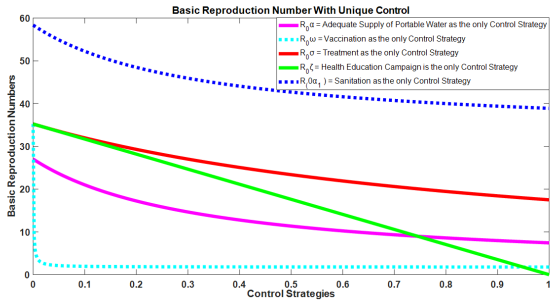

Figure 4. Variations in the effective reproduction number of pathogens when a single control strategy is considered.

From Figures 5 – 8, we observe that there is a significant reduction in disease transmission when two control strategies are considered. Our simulation with bi-controls leads to inequality $\mathcal{R}_{0\alpha\omega} < \mathcal{R}_{0\alpha\sigma} < \mathcal{R}_{0\alpha\alpha_1} < \mathcal{R}_{0\alpha\zeta}$. It is obvious that $\mathcal{R}_{0\alpha\omega}$ is the best-case scenario and it occurs when adequate potable water supply and vaccination are combined a control strategy to eradicate waterborne disease because of a significant decrease in the reproduction number of pathogens. The worst-case scenario, $\mathcal{R}_{0\alpha\zeta}$, occurs when potable water supply and health education campaigns are combined as the only control strategies to limit waterborne diseases in the population. This shows that increasing the number of control strategies will yield a rapid decrease in the reproduction number of pathogens that cause waterborne diseases.

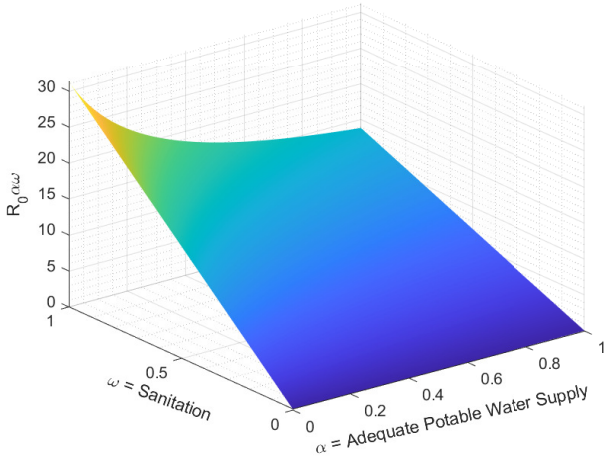

Figure 5. Variations in the effective reproduction number when two control intervention strategies, (i.e., adequate potable water supply) and vaccination are combined.

In Figure 9, we combined three control strategies and observe an improved reduction in the reproduction number of pathogens than using two control strategies. In Figure 10, we combined four control strategies (adequate supply of potable water, vaccination, treatment and health education campaign) leading to the inequality $\mathcal{R}_{0\alpha\omega\sigma\zeta} < \mathcal{R}_{0\alpha\omega\sigma\alpha_1}$. It is

$$\mathcal{R}_0^{eff} = \psi\left(\frac{\mu(1-\zeta)+\omega(1-\eta)}{\mu(\mu+\omega+\varepsilon)}\right)\left[\frac{\beta_1(\gamma+\alpha)(\xi+\alpha_1)+\beta_2(\phi(\xi+\alpha_1)+\pi\xi)+\beta_3\pi(\gamma+\alpha)}{(\delta+\mu+\rho+\sigma)(\gamma+\alpha)(\xi+\alpha_1)}\right] \tag{67a}$$

$$\mathcal{R}_0 = \psi\left[\frac{\beta_1\xi\gamma+\beta_2\xi(\phi+\pi)+\beta_3\pi\gamma}{\mu\xi\gamma(\delta+\mu+\rho)}\right]\Bigg|_{(\zeta,\,\sigma,\,\omega,\,\alpha,\,\alpha_1)=(0,0,\,0,\,0,\,0)} = 217.8960 \tag{67b}$$

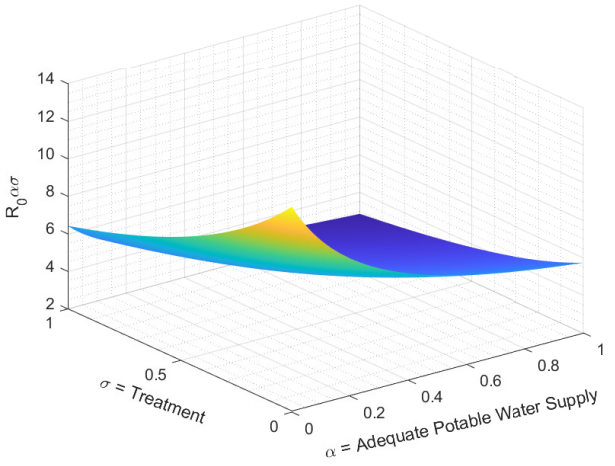

Figure 6. Variations in the effective reproduction number of pathogens when two control intervention strategies (adequate potable water supply and treatment) are combined.

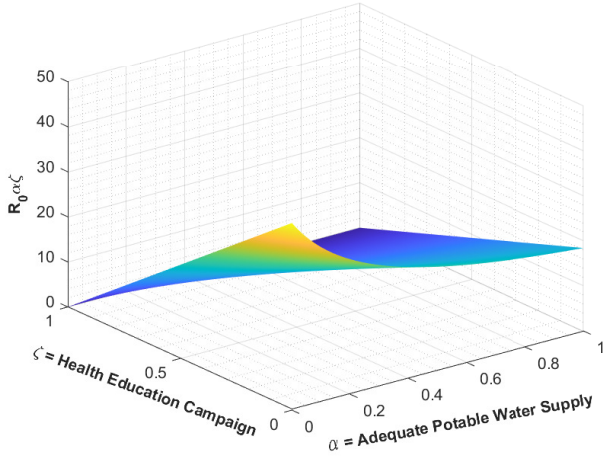

Figure 7. Effective reproduction number of pathogens when two control strategies (i.e., adequate potable water supply and health education campaign) are combined.

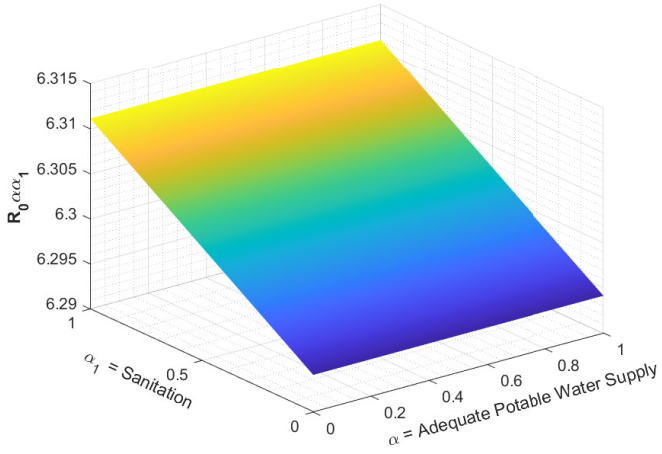

Figure 8. The effective reproduction number of pathogens when two control intervention strategies (i.e., adequate potable water supply and sanitation) are combined.

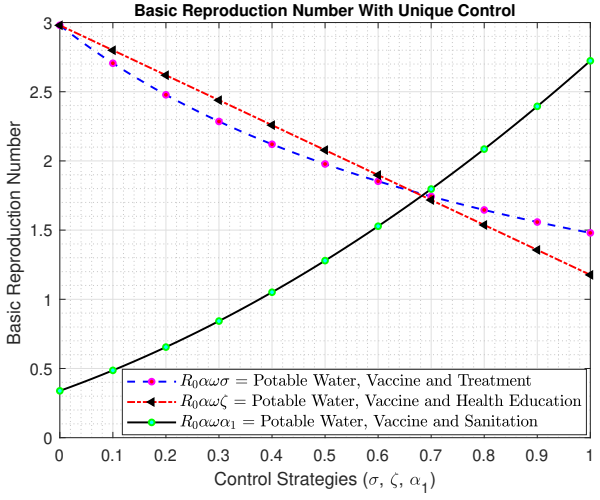

Figure 9. Variations in the effective reproduction number when three control intervention strategies are combined.

obvious that $\mathcal{R}_{0\alpha\omega\sigma\zeta}$ is the best-case scenario. Thus, increasing the number of control strategies significantly reduces the reproduction number of pathogens, so waterborne diseases will not be endemic in the population.

We considered the optimal system under conditions where no control measures were implemented—specifically, without a health education campaign ($\tau_1 = 0$), vaccination ($\tau_2 = 0$), treatment ($\tau_3 = 0$), adequate supply of potable water ($\tau_4 = 0$), and sanitation ($\tau_5 = 0$). The results, shown in Figure 11, reveal a significant increase in both the number of infected individuals and the pathogen population in water reservoirs and dumpsites.

We also simulated the optimal system with all control measures in place—health education campaign ($\tau_1 \neq 0$), vaccination ($\tau_2 \neq 0$), treatment ($\tau_3 \neq 0$), adequate supply of potable water ($\tau_4 \neq 0$), and sanitation ($\tau_5 \neq 0$) as illustrated in Figure 12. The results demonstrate a significant reduction in the number of infected individuals and the pathogen population in water reservoirs and dumpsites, with these numbers approaching zero. This trend is reflected in

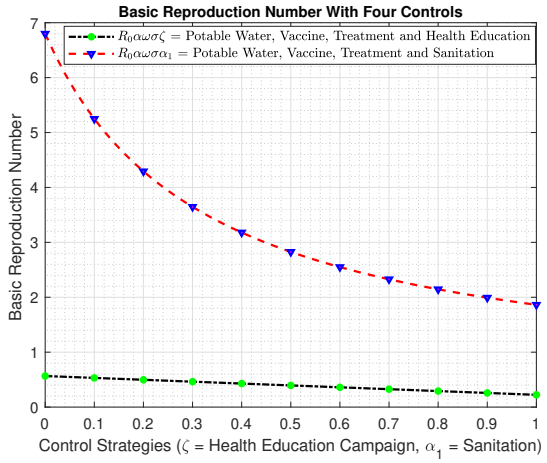

Figure 10. Variations in the effective reproduction number when four control intervention strategies are combined.

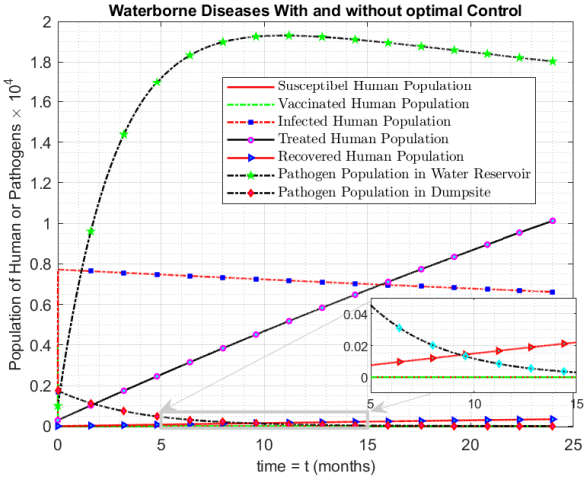

Figure 11. The waterborne disease impacts in human communities (susceptible, vaccinated, infected, treated, recovered) and pathogenic communities (reservoir and dumpsite) when there are no optimal controls (i.e., $\tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 = 0$.

the increased number of individuals treated for waterborne diseases and who subsequently recover. From Figures 13-15, we applied optimisation models to address the dynamics of infected human populations, pathogen concentrations in water reservoirs and dumpsites, and the population of individuals who have recovered from infection. Figure 13 highlights the critical impact of control measures on pathogen levels in water reservoirs, with a remarkable reduction of 19,274.64 pathogens prevented. Similarly, Figure 14 demonstrates the significance of control interventions in mitigating pathogen proliferation in dumpsites, surpassing the uncontrolled scenario. Lastly, Figure 15 shows a notable increase in the rate of recovery among infected individuals when employing an optimal combination of interventions, including vaccination, treatment, potable water supply, sanitation, and health campaigns, compared to scenarios without control measures.

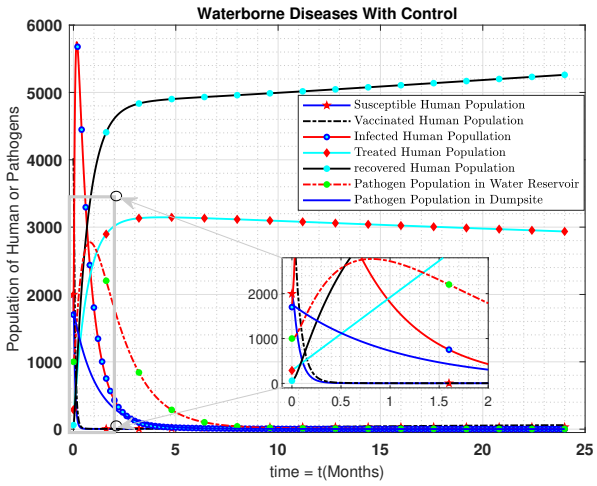

Figure 12. The waterborne disease impacts in human communities (susceptible, vaccinated, infected, treated, recovered) and pathogenic communities (reservoir and dumpsite) when there are optimal controls (i.e., $\tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 \neq 0$).

These findings emphasise the efficacy and practical significance of multifaceted intervention strategies in disease control and environmental health management. Finally, we used a greedy algorithm to select the most promising controls, with the results presented in Tables 4 to 8.

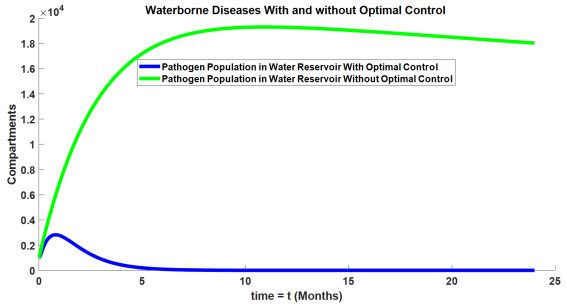

Figure 13. Optimal Control Strategies for Pathogenic Concentration in Water Reservoir.

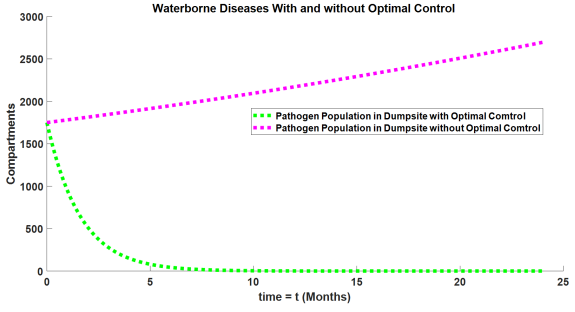

Figure 14. Optimal Control Strategies for Pathogenic Concentration in Dumpsites scenario.

Table 8 shows that the best-case scenario involves combining all the control intervention strategies, including an adequate supply of potable water, vaccination, Treatment, proper sanitation, and a health education campaign.

Table 4: Percentage Reduction in the number of infected human and Pathogen population in water reservoirs and dumpsite when a single control intervention strategy is considered.

	Infected Population			Pathogen Population in Water Reservoir			Pathogen Population in Dumpsite		
Control Strategy	With Control	Without Control	% Reduction	With Control	Without Control	% Reduction	With Control	Without Control	% Reduction
Vaccinated (ω)	15.54	7233.68	99.78	733.31	19279.09	96.20	117.61	117.61	0.00
Treatment (σ)	9.39	7233.68	99.87	538.52	19279.09	97.21	117.61	117.61	0.00
Potable Water Supply (α)	14.23	7233.68	99.80	90.85	19279.09	99.53	117.61	117.61	0.00
Adequate Sanitation (α_1)	15.84	7233.68	99.78	733.30	19279.09	96.20	0.22	117.61	99.83
Health Education Campaign (ζ)	14.31	7233.68	99.80	729.34	19279.09	96.22	117.61	117.61	0.00

Table 5: Percentage Reduction in the number of infected human and Pathogen population in water reservoirs and dumpsites when two control intervention strategies are combined.

	Infected Population			Pathogen Population in Water Reservoir			Pathogen Population in Dumpsite		
Control Strategy	With Control	Without Control	% Reduction	With Control	Without Control	% Reduction	With Control	Without Control	% Reduction
$\alpha\zeta$	7.72	7233.68	99.89	84.45	19279.09	99.56	117.61	117.61	0.00
$\alpha\sigma$	8.84	7233.68	99.88	78.63	19279.09	99.59	117.61	117.61	0.00
$\alpha\omega$	14.74	7233.68	99.80	91.04	19279.09	99.53	117.61	117.61	0.00
$\alpha\alpha_1$	11.61	7233.68	99.84	22.73	19279.09	99.88	0.21	117.61	99.83
$\zeta\omega$	13.46	7233.68	99.81	488.52	19279.09	97.47	117.61	117.61	0.00
$\zeta\sigma$	9.15	7233.68	99.87	290.93	19279.09	98.47	117.61	117.61	0.00
$\zeta\alpha_1$	13.43	7233.68	99.81	483.77	19279.09	97.49	0.21	117.61	99.83
$\sigma\omega$	9.39	7233.68	99.87	537.47	19279.09	97.21	117.61	117.61	0.00
$\sigma\alpha_1$	9.30	7233.68	99.87	291.10	19279.09	98.49	0.21	117.61	99.83
$\omega\alpha_1$	15.48	7233.68	99.79	488.98	19279.09	97.46	0.21	117.61	99.83

In Figure 16, we demonstrate the results of varying the fractional order of the model. It shows that for larger values of fractional order (q), the infected population decays while the recovered population increases. Since the combination of all the control intervention strategies, which includes an adequate supply of potable water, vaccination, Treatment, proper sanitation and health education campaign, is the best-case scenario, we notice that these variables S, V, I, W and D

Table 6: Percentage Reduction in the number of infected human and Pathogen population in water reservoirs and dumpsites when three control intervention strategies are combined.

	Infected Population			Pathogen Population in Water Reservoir			Pathogen Population in Dumpsite		
Control Strategy	With Control	Without Control	% Reduction	With Control	Without Control	% Reduction	With Control	Without Control	% Reduction
$\alpha\zeta\sigma$	4.18	7233.68	99.94	74.26	19279.09	99.61	117.61	117.61	0.00
$\alpha\zeta\omega$	7.75	7233.68	99.89	84.71	19279.09	99.56	117.61	117.61	0.00
$\alpha\zeta\alpha_1$	2.21	7233.68	99.97	13.36	19279.09	99.93	0.21	117.61	99.83
$\alpha\sigma\omega$	8.82	7233.68	99.88	78.68	19279.09	99.59	117.61	117.61	0.00
$\alpha\sigma\alpha_1$	5.12	7233.68	99.93	9.79	19279.09	99.95	0.21	117.61	99.83
$\alpha\omega\alpha_1$	11.63	7233.68	99.84	22.95	19279.09	99.88	0.21	117.61	99.83
$\zeta\sigma\omega$	8.41	7233.68	99.88	537.26	19279.09	97.21	117.61	117.61	0.00
$\zeta\sigma\alpha_1$	7.33	7233.68	99.90	288.80	19279.09	98.50	0.21	117.61	99.83
$\zeta\omega\alpha_1$	13.46	7233.68	99.81	488.62	19279.09	92.47	0.21	117.6096	99.83
$\sigma\omega\alpha_1$	9.30	7233.68	99.87	293.05	19279.09	98.48	0.21	117.6096	99.83

Table 7: Percentage Reduction in the number of infected human and Pathogen population in water reservoirs and dumpsite when Four control intervention strategies are combined together.

	Infected Population			Pathogen Population in Water Reservoir			Pathogen Population in Dumpsite		
Control Strategy	With Control	Without Control	% Reduction	With Control	Without Control	% Reduction	With Control	Without Control	% Reduction
$\alpha\zeta\sigma\omega$	4.19	7233.68	99.94	74.34	19279.09	99.61	117.61	117.61	0.00
$\alpha\zeta\sigma\alpha_1$	0.49	7233.68	99.99	5.40	19279.09	99.97	0.21	117.61	99.83
$\alpha\zeta\omega\alpha_1$	2.27	7233.68	99.97	13.66	19279.09	99.93	0.21	117.61	99.83
$\alpha\sigma\omega\alpha_1$	5.11	7233.68	99.95	9.85	19279.09	99.95	0.21	117.61	99.83
$\zeta\sigma\omega\alpha_1$	7.34	7233.68	99.90	291.34	19279.09	98.49	0.21	117.61	99.83

Table 8: Percentage Reduction in the number of infected human and Pathogen population in water reservoirs and dumpsites when Five control intervention strategies are combined together.

	Infected Population			Pathogen Population in Water Reservoir			Pathogen Population in Dumpsite		
Control Strategy	With Control	Without Control	% Reduction	With Control	Without Control	% Reduction	With Control	Without Control	% Reduction
$\alpha\zeta\sigma\omega\alpha_1$	0.34	7233.68	99.99	4.46	19279.09	99.98	0.21	117.61	99.83

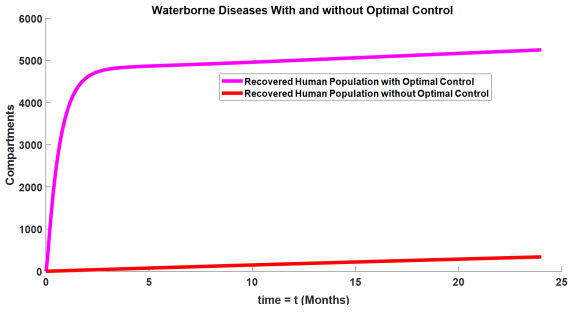

Figure 15. Optimal Control Strategies for Recovered Human Dynamics.

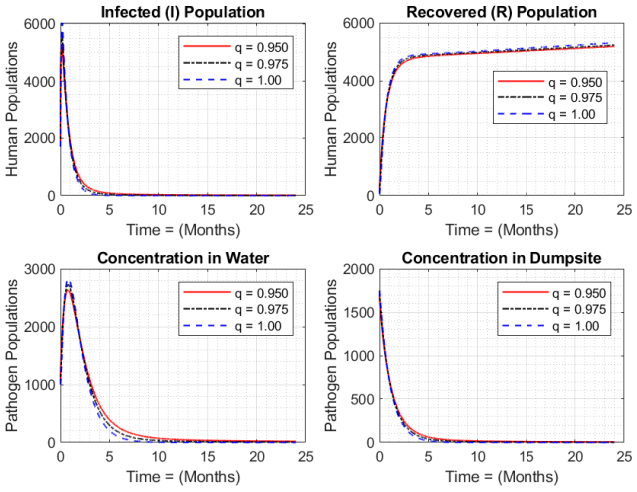

Figure 16. Effects of fractional order on the smart waterborne disease control

decay faster with an increase in the fractional order value (q). By improving the efficiency of the control strategies in the population, the spread of waterborne diseases will be effectively controlled in less time. When non-integer values of the fractional parameter, α, are used, the compartments of the proposed models offer excellent feedback, and the increase or decrease takes place faster in small fractional orders than in large fractional orders. Fractional-order derivations are the most compelling and reliable substitute; they have been demonstrated to be more effective than classical orders in explaining physical processes. In essence, the non-integer value of (q) determines the speed of the compartmental dynamics, allowing fractional order models to capture a wide range of dynamics, from fast and rapid changes to slow and gradual ones, making them versatile tools for modelling complex systems.

Figures 3 and 4 showed, respectively, the effect of individual control strategies using the effective reproduction number of the model, that is, no control is consideration versus control consideration. Figure 4 – 8 represent the combined effect of control strategies on the basic reproduction number of the model with the greatest positive effect being felt $\mathcal{R}_{0\alpha\omega}$ which is when combination of adequate potable water supply and vaccination as control strategy to eradicate water borne diseases is employed. From Figures 9 and 10, it is seen that $\mathcal{R}_{0\alpha\omega\sigma\zeta}$ is the best-case scenario which is combination of adequate supply of potable water, vaccination, Treatment and health education campaign. The combination of the four control strategies considerably diminishes the reproduction number and as such, waterborne diseases will not be widespread in the population. Thus, increasing the number of controls produces a quick decline in the reproduction number. This means that waterborne disease can only be eliminated from the population if the control measures are effective. The performance rates of the control strategies to the dynamics of waterborne disease were further shown in Figures 11 and 12. It will be observed that control strategies can positively affect the dynamics waterborne disease, i.e., the more are effective and efficient, the fewer the population

that will be infected with waterborne disease. Figures 13 and 14 illustrates the effect of optimal control on the population of pathogen both in water reservoir and dumpsite. It was observed that there is a significant decrease in the dynamics of the pathogen both in water reservoir and dumpsite. The result indicated that combination of the control strategies plays a very important role in the prevention and eradication of waterborne disease in the population. Consequently, it will be advisable to shrink contact between the infected and susceptible, infected water reservoir and susceptible and infected dumpsite and susceptible to the barest minimum. Figure 15 shows that there is a larger recovery rate of the infected individuals with the optimal combination of campaign, vaccination, treatment, adequate supply of potable water and sanitation than when there is no control.

We used a greedy algorithm to select the most promising controls. Table 8 shows that combining all the control intervention strategies—adequate potable water supply, vaccination, treatment, proper sanitation, and health education campaigns—is the best-case scenario.

5.3. Findings From Numerical Simulation Results and Greedy Algorithm

A suitable seven (7) compartmental Caputo fractional order Biodynamic model is used to demonstrate the impact of adequate potable water supply and consumption, health education campaign, sanitation, vaccination, and treatment on the control of waterborne diseases. Increasing the rate of supply and consumption of potable water, proper sanitation and vaccination will probably result in the reduction of individuals infected with waterborne diseases. Therefore, intensifying efforts in providing potable water and educating the population on the importance of its consumption and proper hygiene (sanitation) will greatly minimise the spread and circulation of waterborne diseases within the population and this is demonstrated in figures above. We therefore provide forecasts that when the basic reproduction number of waterborne diseases in the presence of the control strategies of waterborne diseases is below unity ($\mathcal{R}_0^{eff} < 1$), then it can be eliminated within the population.

From the numerical analysis of the Biodynamic model, it was discovered that:

i. An effective combination of all the control strategies of susceptible individuals together with robotic identification of the infected can reduce the population of humans infected with waterborne diseases and also minimise the population of pathogens in water reservoirs and dumpsites.
ii. By effectively combining all the control strategies, we have the following:
    - The total number of infections averted is 7253.34.
    - The number of pathogens averted from the water reservoir is 19274.64 and
    - The population of pathogens in dumpsites without control is greater than the population when there are controls and the number of pathogens averted from the dumpsite is 117.40.

From Tables 4 to 8 and the greedy algorithm, we observe that combining all the control intervention strategies is the most effective and efficient method for eradicating waterborne diseases in the population.

The results indicate that various factors contribute significantly to the reduction of waterborne diseases. Here are some specific effects:

- Adequate potable water supply and consumption: This reduces the risk of consuming contaminated water, thereby preventing diseases like cholera, typhoid, and dysentery.
- Health education campaigns: These initiatives educate people about the importance of clean water and proper sanitation practices, thereby aiding disease prevention efforts.
- Sanitation: Implementing proper sanitation practices, such as locating latrines away from water sources, can prevent water contamination and decrease the transmission of diseases like diarrhoea and hepatitis.
- Vaccination: Vaccines offer immunity against specific waterborne diseases, such as cholera and typhoid, and are particularly beneficial in regions where such diseases are prevalent.
- Treatment: Effective treatment of waterborne diseases helps prevent infected individuals from spreading the illness to others, thereby reducing overall disease transmission.

6. Conclusion

In this paper, we proposed a biodynamic model for smart waterborne diseases management in the presence of digital technologies in an unplanned setting where human populations vary with time. The model uses fractional order derivative to determine optimal control conditions of pathogens in water reservoir and dumpsites within a scalable population under the scalable advantages of LoRa IoT network. Five control parameters (vaccination, treatment, potable water supply, adequate sanitation, and health education campaigns) are considered. Our solution is proven to be bounded and showed that the disease-free equilibrium is locally asymptotically stable if the rate of reproduction $\mathcal{R}_0^{eff} < 1$ and unstable if $\mathcal{R}_0^{eff} > 1$. By transmitting the five control parameters over the IoT network, the percentage reduction in infected human populations and pathogen populations in water reservoirs and dumpsites are determined. With individual parameters, the model showed slight impacts when compared with hybrid parameters. For instance, vaccination achieves a 15.54% reduction in infected human populations and a 99.78% reduction in pathogen populations in water reservoirs. When combined, our results show that, the combination of potable water supply and health education achieves a 7.72% reduction in infected human populations and a 99.89% reduction in pathogen populations in water reservoirs with IoT interventions. Moreover, incorporating three or more strategies leads to even higher reductions. Similarly, the combination of all five intervention strategies results in a remarkable 0.34% reduction in infected human populations and a 99.56% reduction in pathogen populations in water reservoirs. Across all combinations, the most effective strategies include potable water supply,

treatment, vaccination, adequate sanitation, and health education campaigns. These results highlight the importance of a comprehensive, multi-faceted approach to controlling infection spread and pathogen proliferation in both human populations and environmental reservoirs. The study suggested that controlling and preventing waterborne diseases could be achieved by improving adequate potable water supply and consumption, health education campaign, sanitation, vaccination, and treatment. This is because adequate water supply and consumption will reduce transmission of the disease vectors from water to person, adequate sanitation will prevent transmission from dumpsites to person, person to person, at the same time prevent the dumpsites from being highly contagious and transmission from dumpsite to water reservoir. Lastly, effective vaccination of the susceptible population and treatment of the infected population decreases person to person transmission and the rate at which water reservoirs are being contaminated. It follows that by combing these control strategies, waterborne diseases will be significantly minimised in the population.

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In most developing countries, people heavily rely on water from boreholes due to the adequate supply provided under the right environmental conditions. The first phase of this research involved drilling an automated borehole designed to alleviate water scarcity in Iji-Nike, the host community. The borehole drilling project was completed and commissioned with support from a grant by the Institute of Electrical and Electronics Engineers (IEEE) under the project titled "Automated Borehole Design and Massive Rural Health Education in Iji-Nike Community in Enugu State (22-HAC-119)."

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A. Appendix A

The following is a pseudocode of the MATLAB implementation of the biodynamic model. For brevity, we define $\mathcal{F}(n) = [t(n), S(n), V(n), I(n), T(n), R(n), W(n), D(n)]$, $r_1(n) = ((n+1-k).^a. * (n-k+2+a) - (n-k).^a. * (n-k+2+2*a))$, $r_2(n) = ((n+1-k).^(a+1) - (n-k).^a. * (n-k+1+a))$, $\mathcal{F}(k) = [t(k), S(k), V(k), I(k), T(k), R(k), W(k), D(k)]$ and $\mathcal{F}(k-1) = [t(k-1), S(k-1), V(k-1), I(k-1), T(k-1), R(k-1), W(k-1), D(k-1)]$, the pseudocode becomes:

$$\mathrm{S(n+1) = S(1) + ((1-a)/abc) * f1(\mathcal{F}(n)) + (a/abc). * (h^a/\gamma(a+2)). * sum(r_1(n). * f1(\mathcal{F}(k)) - r_2(n). * f1(\mathcal{F}(k-1)));}$$
$$\mathrm{V(n+1) = V(1) + ((1-a)/abc) * f2(\mathcal{F}(n)) + (a/abc). * (h^a/\gamma(a+2)). * sum(r_1(n). * f2(\mathcal{F}(k)) - r_2(n). * f2(\mathcal{F}(k-1)));}$$
$$\mathrm{I(n+1) = I(1) + ((1-a)/abc) * f1(\mathcal{F}(n)) + (a/abc). * (h^a/\gamma(a+2)). * sum(r_1(n). * f1(\mathcal{F}(k)) - r_2(n). * f1(\mathcal{F}(k-1)));}$$
$$\mathrm{T(n+1) = T(1) + ((1-a)/abc) * f1(\mathcal{F}(n)) + (a/abc). * (h^a/\gamma(a+2)). * sum(r_1(n). * f1(\mathcal{F}(k)) - r_2(n). * f1(\mathcal{F}(k-1)));}$$
$$\mathrm{R(n+1) = R(1) + ((1-a)/abc) * f1(\mathcal{F}(n)) + (a/abc). * (h^a/\gamma(a+2)). * sum(r_1(n). * f1(\mathcal{F}(k)) - r_2(n). * f1(\mathcal{F}(k-1)));}$$
$$\mathrm{W(n+1) = W(1) + ((1-a)/abc) * f1(\mathcal{F}(n)) + (a/abc). * (h^a/\gamma(a+2)). * sum(r_1(n). * f1(\mathcal{F}(k)) - r_2(n). * f1(\mathcal{F}(k-1)));}$$
$$\mathrm{D(n+1) = D(1) + ((1-a)/abc) * f1(\mathcal{F}(n)) + (a/abc). * (h^a/\gamma(a+2)). * sum(r_1(n). * f1(\mathcal{F}(k)) - r_2(n). * f1(\mathcal{F}(k-1)));}$$

where $f_i = f_i\big(t, S(t_i), V(t_i), I(t_i), T(t_i), R(t_i), W(t_i), D(t_i)\big)$, $i = 1, 2, \cdots, 7$.

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