

## **Formal methods for division: Evaluating the benefits of pre-teaching mathematics using a ‘flipped classroom’ approach**

Lorna Earle and Caroline Rickard

*University of Chichester*

This paper briefly outlines our research into supporting trainee teachers in their quest to understand formal methods for division, often considered a tricky topic to understand deeply and to teach effectively. Literature suggested that providing pre-teaching, using a ‘flipped classroom’ approach, might be beneficial and initial findings from our research suggest this to be the case.

**Keywords: division; formal written methods; primary; initial teacher education; flipped classroom**

### **Literature supporting the idea of a ‘flipped classroom’**

Lake et al.’s (2016) meta-analysis exposes the weak mathematical foundations of many undergraduate university students, highlighting the importance of, and “urgent need” (p.2) for, appropriate mathematical interventions, particularly in the first year of university courses. One such intervention could be through a ‘flipped classroom’ approach. Johnston (2017) explored the potential of this for delivering a university numerical methods mathematics course. She explains that the ‘flipped classroom’ approach requires students to have access to material prior to the class which in turn allows them to focus on processing learning in class; for mathematics courses the basic course material of definitions, facts and skills should be introduced before the session through instructional videos. Johnston’s (2017) findings build on and concur with previous research projects (such as Love, Hodge, Grandgenett & Swift, 2014 and Loch, Jordan, Lowe & Mestel, 2014) that students like the use of instructional videos prior to sessions and the approach has been found to have a positive effect on student performance.

Vejar and Raspopovic’s (2015) research also found that undergraduate mathematics students’ confidence in specific topics was greatly influenced by whether the students had watched the provided pre-session material prior to attending a taught session. They concluded that using the ‘flipped classroom’ approach enabled sessions to be active learning opportunities in which the students stepped out of their comfort zone, with evidence showing students’ belief that engagement with the online resources directly correlated to an enhanced learning experience. Lake et al. (2016) also concluded that early intervention programmes which directly addressed particular mathematical skills deficits were successful and should be considered for use by universities offering courses involving mathematics. We wanted to consider, therefore, whether any aspects of this approach could be particularly beneficial for primary trainee teachers. Key themes were identified to form the basis of the research project: the students’ relationship with the topic (division); their confidence and understanding of the related formal methods, both procedural and conceptual; their confidence to participate in taught university sessions; and their confidence to teach division.

## **Context and methods**

The research project took place during a year one curriculum mathematics module on our BA (Hons) Primary Teaching degree. Prior to the start of the module, students completed a subject knowledge audit and from this developed a personal action plan. Review of these action plans showed that, as with previous cohorts, there exist two main areas of concern in terms of students' subject knowledge and confidence: division and fractions. The concerns the students identified were more focused within division, with 38 of the 104 students in the cohort identifying formal written methods for division as a topic they lacked confidence in. As a result the division session of the first year module was chosen as the focus of the research project, offering a more specific opportunity to evaluate the success of a pre-teaching intervention. Two video clips were used for this. In the first a group of Y5 children were supported in their understanding of short division using Dienes apparatus. The second was aimed directly at the trainee teachers, building on the method seen in the first video and modelling the long division procedure. In addition there was some prescribed pre-session reading.

A mixed methods approach involving questionnaires and semi-structured interviews was used to gather data for this study, enabling the collection of both quantitative and qualitative data. Triangulation of these findings allowed a more complete understanding of the data than either method could have afforded if used independently. Fifty-nine students returned their completed questionnaire, a response rate of 57%. Fifteen students volunteered to take part in the follow up interviews, of whom six were selected. Interviewees were chosen as they displayed interesting characteristics, particularly relevant to the themes of this study, warranting further investigation. These characteristics were shared by others in the cohort, so the responses allowed for a better depth of understanding of questionnaire findings.

## **Findings**

### ***Relationship with division***

The initial section of the questionnaire, completed prior to engaging with the video clips, began by assessing the participants' relationship with division. A deliberately ambiguous, open question was asked; 'write down the first three words that come into your mind when you think of division.' This question was designed to be one that could be interpreted in different ways by different people so as to allow the participants freedom in their interpretation and response. Just ten blank spaces were left across seven students' responses, giving a total of 167 words. 138 of the words were content based, twenty-two were negative, six exhibited possible misconceptions and there was one positive word. Thirteen participants in total wrote at least one negative word. Of the twenty-two negative words, twelve came from just four participants, who gave entirely negative responses.

Interviewee one was chosen as they had given an entirely negative response ('complicated', 'confusing' and 'difficult'). They explained that they had had a negative experience in school and they still held negative feelings towards division because of this, considering it to be a result of the teaching they had experienced. This exemplifies two conclusions drawn by Bekdemir (2010); that teachers can be responsible for the creation of maths anxiety and that there is a strong possibility that trainee teachers who lack confidence in their own abilities will have negative emotions towards mathematics. Bekdemir (2010) insists it is incredibly important to

“break this cycle” (p.313) as these negative emotions may be passed on to their pupils. Discussing the matter further with interviewee one, they explained that they, “would choose more positive words following the videos.” These comments suggest the use of the ‘flipped classroom’ may be a possible approach to begin to ‘break the cycle’. This is supported by the research of Lesseig and Krouss (2016) who state that students who have low motivation or may have had previous unsuccessful experiences with the course material may find the ‘flipped classroom’ approach particularly beneficial.

**Assessing confidence**

Seven-point Likert scales were then used to ascertain the participants’ feelings towards a range of statements, repeated at the end of the intervention (following engagement with the video clips and reading). Participants were asked to assess their confidence, ‘with 1 being very lacking in confidence and 7 being very confident.’ This approach was taken, as although the interpretation of the Likert scale may vary (Bell, 2010), it would not only give a general indication of the level of participants’ confidence, but more importantly indicate any changes in confidence for individuals.

**Procedural and conceptual confidence**

The research focused on formal methods for division. The first statement required the students to rank their confidence in relation to the procedures whereas the second statement required the participants to rank their perceived conceptual understanding. Figure 1 shows the general increase in confidence rankings after completing the pre-session reading and watching the two videos.

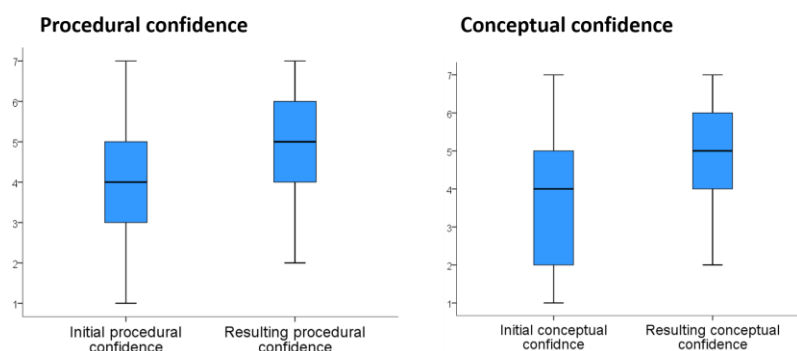


Figure 1: Participants’ responses when assessing their confidence and understanding of the formal methods for division, both procedural and conceptual, before and after completing the ‘flipped classroom’ pre-teaching intervention. 1 – very lacking in confidence, 7 – very confident.

In light of this general increase, it was interesting to investigate how individual rankings had changed.

Procedural confidence:

Number of scale points changed	+4	+3	+2	+1	0	-1	-2
Number of participants	0	5	11	34	7	2	0

Median increase: 1 scale point, 85% report an increase in confidence

Conceptual understanding:

Number of scale points changed	+4	+3	+2	1	0	-1	-2
Number of participants	1	5	8	32	12	0	1

Median increase: 1 scale point, 78% report an increase in confidence

Figure 2: Individual changes in confidence rankings when assessing procedural confidence and confidence in conceptual understanding following the ‘flipped classroom’ pre-teaching intervention.

Of the seven participants who reported no change for procedural confidence, all started by ranking themselves as confident, as did all but two of the twelve who reported no change in their perceived conceptual understanding.

*Definitions, facts and skills*

The videos had been designed to focus on developing procedural confidence and conceptual understanding, in line with Johnston’s (2017) suggestion that definitions, facts and skills should be introduced before a session through instructional videos. All six interviewees agreed that facts and definitions were key, with interviewee five explaining that giving definitions helps as it “makes you more aware of things in the session, you understand what they mean.” However, there was a split in opinion whether skills should be introduced in the videos or in session. Four of the six expanded their answer in relation to skills, with three agreeing that they should be taught through the videos, emphasising that knowing these then gave them confidence to participate, feeling that they better understood why the methods worked and how to get to an answer. Interviewee six (whose confidence decreased after the videos) disagreed, explaining, “skills are good to mention but easier to learn in a classroom with people” which supports one of Loch et al.’s (2014) hesitations regarding the use of this approach as it is potentially a passive experience for the learners.

*Confidence to participate*

The questionnaire data shows that most of the participants felt more confident to participate in the session following the pre-teaching intervention. The median increase in confidence ranking was 1 scale point. All six interviewees highlighted confidence to participate in the session as the main benefit of the pre-teaching interventions. Interviewee six added that the videos “give you more confidence to ask questions in session as you know you will be able to explain the ideas, instead of staying silent.” Interviewee six’s response supports the findings of Vejar and Raspopovic (2015) who found that students notably asked for more help after they implemented a flipped classroom approach.

Interviewee two had specifically been selected as they started with the lowest confidence to participate out of the volunteers. Their confidence increased following the videos, and they explained they were less worried about making errors in the session having first addressed their subject knowledge. They strongly advocated the use of videos as they explained, “the discussions in session are sometimes a bit overwhelming, it’s better to have had time to process individually then your brain doesn’t just shut down when you don’t feel confident in session!”

*Confidence to teach formal methods for division*

The final statement required the participants to rank their confidence to teach formal methods for division. Of the nine participants who maintained the same confidence score, only three initially lacked confidence to teach division.

Confidence to teach formal methods for division:

Number of scale points changed	+4	+3	+2	+1	0	-1	-2
Number of participants	0	5	22	21	9	2	0

Median increase: 1 scale point, 81% report an increase in confidence

Figure 3: Individual changes in confidence ranking when assessing confidence to teach formal methods for division following the ‘flipped classroom’ pre-teaching intervention.

This idea was explored in more depth in the follow-up interviews. Five of the six interviewees considered the videos to have particular benefits in relation to this; interviewee six was more hesitant however, they did suggest that it was helpful to have the exemplification of the ideas in an actual classroom in the first video. From the other five interviewees there was general agreement that the benefits of the videos included being able to see how someone teaches division, how they explain it, breaking it down step by step and the resources that can be used. This in turn increases confidence to teach, especially as the videos can be referred to whilst on teaching placement. The interviewees surmised that even for people who already feel confident with the subject knowledge, it is beneficial to have an example of how it can be taught. An important theme that emerged was that if more of the mathematics was learnt through the video, more time could be spent on pedagogy in sessions, suggesting, as Fulton (2012) proposes, that the flipped classroom approach allows classroom time to be used more effectively.

Interviewee three was selected as they showed a much lower confidence to teach than any other response to the statements (ranked at point 2). After the videos their confidence increased by three scale points. Interviewee three stated that they found the videos “massively helpful.” They explained that they had never been taught formal methods for division so felt inadequate, even with an A grade at GCSE.

### *Evaluation of the videos*

Having analysed the findings it was evident that the participants’ confidence had generally increased in the focus areas. The final questions of the questionnaire were designed to ascertain the views of the participants in relation to the usefulness of the videos specifically. Worded responses were used rather than a numerical scale. Participants were first asked, ‘to what extent do you think the videos helped develop your subject knowledge in preparation for the session?’ 97% of the participants indicated that they had found the videos helpful, with 80% describing them to have helped either ‘a lot’ or ‘a huge amount’. Participants were then asked, ‘how useful do you think [the videos] were?’ Again, 97% considered them to be useful, with 67% describing them as either ‘very useful’ or ‘incredibly useful’. Of the interviewees, five out of six found the flipped classroom approach, specifically the use of the videos, to be beneficial. Their comments included:

It gives us the opportunity to see different approaches and adapt these. Having the videos helps with my confidence as I know they will be available in the future.

I could do it in my own time when I felt awake and as many times as I wanted. If this had been done in the session, it would have knocked my confidence as I would have felt like everyone could do it.

Being able to see it step by step was really helpful. You got a visual and an explanation and you could take notes. Reading wouldn’t be the same.

It was a good approach, you could do bits at home. The different approach means you can think differently. [It meant] you could do more group work in the session.

Interviewee six had been selected as their confidence decreased after watching the videos. They stated that they did not like the use of videos as they could not ask for an explanation of things they did not understand.

Finally, participants were asked if they would like more video tutorials; 81% responded ‘yes’. This is in keeping with Lesseig and Krouss’ (2016) findings that student perceptions of the flipped classroom approach are generally positive.

## Conclusion

In keeping with Lake et al.'s (2016) exposure of an “urgent need” (p.2) for appropriate mathematical interventions, and others’ research recommending the benefits of the ‘flipped classroom’ approach, this study suggests that pre-teaching interventions could prove beneficial within primary teacher training. The confidence of the majority of participants was increased and more pre-session videos requested. Videos will be embedded into the module as part of pre-session tasks. Supporting reading will also be available alongside these for students, such as interviewee six, who do not engage so well with the videos. Consideration has been given to the idea of interviewee three that these could be voluntary, but as the benefits were so widely recognised, they will become an integral part of the pre-session teaching. Having the opportunity to watch and evaluate the pedagogy within the videos was identified as a benefit even if students are already confident with their subject knowledge.

This project mirrors Johnston’s (2017) findings that “while a very small number of students did not like the videos, the vast majority did, with some students commenting very enthusiastically about them” (p.496). Lesseig and Krouss (2016) recognise that significant effort is required to produce flipped classroom materials and that the method is still in its infancy; as such key ideas can be taken forward from this trial and will continue to be evaluated for their effectiveness.

## References

- Bekdemir, M. (2010). The pre-service teachers’ mathematics anxiety related to depth of negative experiences in mathematics classroom while they were students. *Education Studies in Mathematics*, 75, 311-328.
- Bell, J. (2010). *Doing your research project*. 5<sup>th</sup> edn. Maidenhead: Open University Press.
- Fulton, K.P. (2012). 10 reasons to flip. *Phi Delta Kappan*, 94, 20-24.
- Johnston, B.M. (2017). Implementing a flipped classroom approach in a university numerical methods mathematics course. *International Journal of Mathematics Education in Science and Technology*, 48, 485-498.
- Lake, W., Wallin, M., Woolcott, G., Boyd, W., Foster, A., Markopoulos, C., & Boyd, W. (2016). Applying an alternative mathematics pedagogy for students with weak mathematics: Meta-analysis of alternative pedagogies. *International Journal of Mathematics Education in Science and Technology*, 48, 215-228.
- Lesseig, K., & Krouss, P. (2016). Implementing a flipped instructional model in college algebra: Profiles of student activity. *International Journal of Mathematics Education in Science and Technology*, 48, 1-13.
- Loch, B., Jordan, C.R., Lowe, T. W., & Mestel, B.D. (2014). Do screencasts help to revise prerequisite mathematics? An investigation of student performance and perception. *International Journal of Mathematics Education in Science and Technology*, 45, 256-268.
- Love, B., Hodge, A., Grandgenett, N., & Swift, A. W. (2014). Student learning and perceptions in a flipped linear algebra course. *International Journal of Mathematical Education in Science and Technology*, 45, 317-324.
- Vejar, E.S., & Raspopovic, M. (2015). Evaluation of flipped classrooms in undergraduate mathematics courses. Proceedings of 6<sup>th</sup> Conference on e-Learning held at Belgrade Metropolitan University, Serbia.